Reflection and transmission of SH-waves at a corrugated interface between two laterally and vertically heterogeneous anisotropic elastic solid half-spaces

S. K. Tomar[1] and Jatinder Kaur[2]

[1]*Department of Mathematics, Panjab University, Chandigarh-160 014, India*
[2]*Department of Mathematics, Guru Nanak Girls College, Santpura, Yamuna Nagar-135 001, Haryana, India*

This paper is concerned with the reflection and transmission coefficients of SH-waves at a corrugated interface between two anisotropic heterogeneous elastic solid half spaces. Both the half spaces are taken transversely isotropic and laterally and vertically heterogeneous. The Rayleigh's method of approximation is adopted and expressions for reflection and transmission coefficients are obtained in closed form for the first-order approximation of the corrugation. In Rayleigh's method, expressions in boundary conditions containing the function defining the corrugated boundary are expanded in Fourier series and unknown coefficients in the solutions are determined to any given order of approximation in terms of a small parameter characteristic of the boundary. The analytical expressions of these coefficients show that they depend upon corrugation of the interface and are strongly influenced by the anisotropy and heterogeneity of the half-spaces. Numerical computations are performed for the case of a particular corrugated interface: $\zeta = c \cos k^* x$ showing that the effect of heterogeneity on the reflection and transmission coefficients is minimum near the normal incidence and dominance of this effect increases with the angle of incidence. For incident wave striking at $45°$, the effect of the corrugation is found significant on the reflection and transmission coefficients. The maximum effect of transverse isotropy on the reflection and transmission coefficients is observed at normal incidence when the values of the anisotropy parameters are 0.5 and 0.8 for the upper and lower half-spaces, respectively. The effect of frequency of the incident wave is observed on all reflected and refracted waves. The analytical expressions derived by Tomar and Saini (1997), Gupta (1987) and Asano (1960) are obtained as particular cases with our formulation.
Key words: Reflection, transmission, SH-wave, Rayleigh's method, heterogeneity, anisotropy, corrugation.

1. Introduction

Solving the problem of reflection and refraction of seismic waves from discontinuities is of great help to better understand the internal structure of the earth, and to detect valuable materials as well as hydro-carbons beneath the earth surface. Earth heterogeneities such as mountain basin, mountain roots and ore bodies affect the reflection and transmission of seismic waves, thus the study of the reflection and refraction coefficients due to rough surfaces is of great practical importance in Seismology.

A number of researchers have attempted the problem of reflection and refraction of seismic waves at corrugated interfaces using different methods. Sato (1955) investigated the reflection of elastic waves at a corrugated, free surface by using the Rayleigh's method of approximation. Asano (1960, 1961, 1966) also used the Rayleigh's method to address problems of reflection and refraction of elastic waves at a corrugated interface between two uniform elastic half spaces. Abubakar (1962a) and Dunkin and Eringen (1962) used the perturbation method to study the problem of reflection of body waves from a rough surface of a semi-infinite elastic solid. In perturbation method, the amplitudes of su-

perposed scattered plane waves are expressed into sums of terms whose order of magnitudes are proportional to the powers of the amplitude of the rough surface, the curvature of the rough surface should be small everywhere. Later, Abubakar (1962b, c), using the perturbation technique, studied the reflection and refraction of SH-waves at an irregular interface between two uniform elastic solid half spaces. Adams and Chung-Po-Chang (1964) applied Weber integral to solve wave equation and studied the wave propagation phenomenon at an irregular infinite interface. Aki and Larner (1970) investigated the surface motion of a layered medium with irregular interface due to incident plane SH-wave. The effect of surface irregularity on the propagation of waves in an elastic plate was studied by Sumner and Deresiewicz (1972) using the perturbation technique. Some notable references pertaining to the subject of waves on irregular boundaries are Gilbert and Knopoff (1960), Slavin and Wolf (1970), Yamada and Sato (1976), and Gupta (1978).

In most of those investigations, the elastic media considered were homogeneous. Not much work has been done on heterogeneous media with corrugated boundary. The reflection and transmission coefficients of elastic waves from flat boundaries for the case of inhomogeneous media with several types of velocity distributions have been studied by Acharya (1970), Saini and Singh (1977b), Singh *et al.*

(1978), Malhotra *et al.* (1982) among others. Chattopadhyay and Pal (1982) studied the propagation of SH-waves in an inhomogeneous medium with irregular interface lying over an initially stressed elastic half space. Gupta (1987) studied the problem of reflection and transmission of SH-waves in laterally and vertically heterogeneous media at an irregular boundary. She used the Rayleigh's method to study the effect of lateral and vertical heterogeneities on the reflection and refraction coefficients, and gave the mathematical treatment to compute these coefficients for first and second order approximation of the corrugation. Zhang *et al.* (1997) studied seismic waves in a laterally inhomogeneous medium.

Henneke (1972) studied the effect of anisotropy on reflection and refraction of stress wave at a plane boundary between anisotropic media. Daley and Hron (1977) studied reflection and refraction coefficients at a plane interface due to incident P and SV waves in transversely isotropic elastic media. Saini and Singh (1977a) investigated the effect of transverse isotropy on reflection and transmission coefficients of SH-wave at a plane interface between a homogeneous isotropic half-space and a transversely isotropic half-space. Rokhlin *et al.* (1986) studied these coefficients on a plane interface between two generally anisotropic media. The problems of wave propagation from plane boundaries in anisotropic media have also been studied by Musgrave (1960), Keith and Crampin (1977), Mandal (1991), Ruger (1997) among others. Tomar and Saini (1997) studied the reflection and refraction of SH-waves at a corrugated interface between two transversely isotropic elastic half spaces. Recently, Tomar *et al.* (2002) investigated the reflection and transmission coefficients of SH-waves at corrugated interface between transversely isotropic and visco-elastic solid half-spaces.

The earth medium is complex in nature and full of various types of anisotropy and heterogeneity (see Sheriff and Geldart, 1995; Lay and Wallace, 1995; Shearer, 1999). Moreover, discontinuities in the earth are not perfectly flat (in fact, these discontinuities are stochastically irregular up to some extent). This motivated us to formulate the present model. The present work deals with the problem of reflection and refraction of SH-waves at a corrugated interface between two transversely isotropic and laterally and vertically heterogeneous elastic solid half spaces. Rayleigh's method of approximation has been followed to derive the expressions for reflection and transmission coefficients only for first order approximation of the corrugation due to incident SH-waves. The problems discussed by Tomar and Saini (1997), Gupta (1987) and Asano (1960) have been reduced as particular cases of the present problem.

2. Formulation of the Problem and Its Solution

We consider two laterally and vertically heterogeneous anisotropic elastic solid half spaces H_1 and H_2, separated by a corrugated interface. The x-axis and y-axis are on the horizontal plane, whereas the z-axis is pointing vertically downwards. The elastic constants, densities and velocities in media H_i are given by M_i, N_i; ρ_i and β_{v_i}, β_{h_i} ($i = 1, 2$) respectively. The quantities concerning the upper and lower medium will be denoted by subscript 1 and 2, respectively. The equation of the boundary surface (corrugated interface)
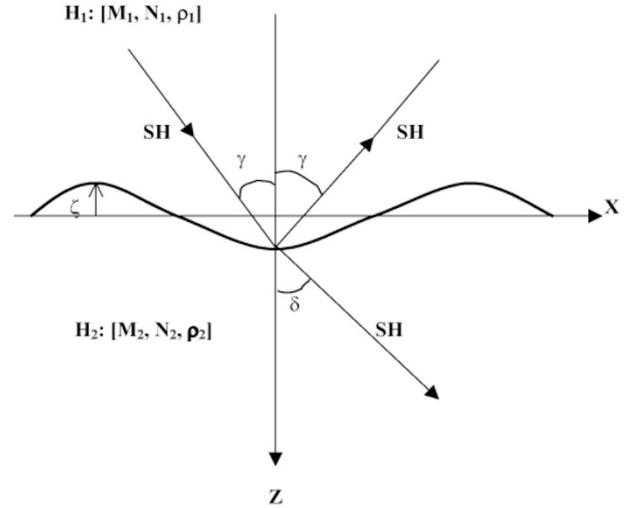
Fig. 1. Geometry of the problem.

is assumed to be

$$z = \zeta \qquad (1)$$

where ζ is a periodic function of x, independent of y, whose mean value is zero. The geometry of the problem is shown in Fig. 1. The Fourier series representation of ζ is given by

$$\zeta = \sum_{n=1}^{\infty} \left[\zeta_n \, e^{\iota n k^* x} + \zeta_{-n} \, e^{-\iota n k^* x} \right], \qquad (2)$$

where ζ_n and ζ_{-n} are Fourier expansion coefficients, n is the series expansion order and $\iota = \sqrt{-1}$. Introducing constants c_1, c_n and s_n as

$$\zeta_1 = \zeta_{-1} = \frac{c_1}{2}, \qquad \zeta_{\pm n} = \frac{c_n \mp \iota s_n}{2}, \qquad (3)$$
$$n = 2, 3, 4, \ldots$$

we obtain

$$\zeta = c_1 \cos k^* x + c_2 \cos 2k^* x + s_2 \sin 2k^* x + c_3 \cos 3k^* x$$
$$+ s_3 \sin 3k^* x + \cdots + c_n \cos n k^* x + s_n \sin n k^* x + \cdots$$

If the interface shape can be expressed by only one cosine term i.e. $\zeta = c_1 \cos k^* x$, the wavelength of corrugation is $2\pi / k^*$, where k^* is the wavenumber.

The equation of motion for SH-wave propagation in a transversely isotropic, inhomogeneous elastic medium with zero body forces can be written as follows:

$$\frac{\partial}{\partial x} \left(N_i \frac{\partial V_i}{\partial x} \right) + \frac{\partial}{\partial z} \left(M_i \frac{\partial V_i}{\partial z} \right) = \rho_i \frac{\partial^2 V_i}{\partial t^2}, \qquad (4)$$

where the constants N_i and M_i and ρ_i are defined earlier and V_i is y-component of the displacement vector in medium H_i. Since the medium is assumed to be laterally and vertically heterogeneous, we shall take

$$\{N_i, M_i, \rho_i\}(x, z) = \{N_{io}, M_{io}, \rho_{io}\} \, p(x) r_i(z), \qquad (5)$$

where N_{i0}, M_{i0} and ρ_{i0} are constants. We assume the solution of Eq. (4) in the form

$$V_i(x, z, t) = \frac{1}{\sqrt{M_i(x, z)}} X(x) Z(z) \exp(\iota \omega t), \qquad (6)$$

where ω is the angular frequency. Substituting (5) and (6) into (4), we get the following differential equation:

$$\left[\frac{d^2 X}{dx^2} - \epsilon^2 X\right] Z = -\frac{M_{i0}}{N_{i0}} \left[\frac{d^2 Z}{dz^2} - \left\{b_i^2 - \frac{\rho_{i0}}{M_{i0}}\omega^2\right\} Z\right], \tag{7}$$

where ϵ and b_i are given by

$$\epsilon^2 = \frac{1}{2p}\frac{d^2 p}{dx^2} - \frac{1}{4}\left(\frac{1}{p}\frac{dp}{dx}\right)^2, \tag{8}$$

$$b_i^2 = \frac{1}{2r_i}\frac{d^2 r_i}{dz^2} - \frac{1}{4}\left(\frac{1}{r_i}\frac{dr_i}{dz}\right)^2.$$

For wave propagating in positive x-direction, let us take $X(x) = \exp(-\iota k x)$, and plugging it into Eq. (7), we obtain

$$\frac{d^2 Z}{dz^2} - \frac{N_{i0}}{M_{i0}}\left(k^2 + \epsilon^2 + \frac{M_{i0}}{N_{i0}}b_i^2 - \frac{\omega^2}{\beta_{h_i}^2}\right) Z = 0, \tag{9}$$

where $\beta_{h_i} = \sqrt{N_{i0}/\rho_{i0}}$. The solution of Eq. (9) can be written as

$$Z = Ae^{-s_i z} + Be^{s_i z}, \tag{10}$$

where A and B are constants and

$$s_i = \sqrt{\frac{N_{i0}}{M_{i0}}\left(\epsilon^2 + k^2 + \frac{M_{i0}}{N_{i0}}b^2 - \frac{\omega^2}{\beta_{h_i}^2}\right)}.$$

Thus the time harmonic solution of Eq. (4) may be written as

$$V_i = \frac{1}{\sqrt{M_{i0}p(x)r_i(z)}}\left[Ae^{-s_i z} + Be^{s_i z}\right]e^{\iota(\omega t - kx)},$$

$k = \omega \sin\theta/\beta_h$ is the horizontal component of the wave number given by Gupta (1965), $\beta_h = \sqrt{N_0/\rho_0}$ and θ is the angle between the normal to the wavefront and the positive direction of z-axis.

Equations in (8) may be written as

$$\frac{d}{dp}\left\{\frac{1}{p}\left(\frac{dp}{dx}\right)^2\right\} = 4\epsilon^2, \qquad \frac{d}{dr_i}\left\{\frac{1}{r_i}\left(\frac{dr_i}{dz}\right)^2\right\} = 4b_i^2. \tag{11}$$

On integrating these equations twice, we obtain

$$x + \epsilon_2 = \pm\frac{1}{2}\int \left(\epsilon^2 p^2 + \epsilon_1 p\right)^{-1/2} dp,$$

$$z + \epsilon_4 = \pm\frac{1}{2}\int \left(b_i^2 r_i^2 + \epsilon_3 r_i\right)^{-1/2} dr_i, \tag{12}$$

where ϵ_1, ϵ_2, ϵ_3 and ϵ_4 are constants of integration. In particular, when $\epsilon_1 = \epsilon_2 = 0$, we can obtain the form of $p(x) = \exp(\pm 2\epsilon x)$ and when $\epsilon_3 = \epsilon_4 = 0$, we obtain the form of $r_i(z) = \exp(\pm 2b_i z)$. In case, when $\epsilon = 0$ and $\epsilon_1\epsilon_2^2 = 1$, we obtain $p(x) = (1 + x/\epsilon_2)^2$ and when $b_i = 0$, $\epsilon_3\epsilon_4^2 = 1$, we obtain $r_i(z) = (1 + z/\epsilon_3)^2$. If $\epsilon_1 = \epsilon^2 \neq 0$, then we obtain $p(x) = \sinh^2(x + \epsilon_2)\epsilon$, and when $\epsilon_3 = b_i^2 \neq 0$, we obtain $r_i(z) = \sinh^2(z + \epsilon_4)b_i$ (cf. Singh $et\ al.$, 1976). Thus we see that the solutions of equations in (8) are of the form

$\exp(ax)$, $(1 + ax)^2$ or $\sinh^2 ax$. In the present analysis, we shall take for convenience, the exponential form of $p(x)$ and $r_i(x)$.

Considering an incident SH-wave of unit amplitude, the displacement in the medium H_1 due to both incident and reflected SH-wave, is given by

$$V^{inc+refl} = \frac{1}{\sqrt{M_{10}p(x)r_1(z)}}\left\{e^{-qz} + Be^{qz}\right\}e^{\iota\omega\left(t - \frac{x\sin\gamma}{\beta_{h_1}}\right)}, \tag{13}$$

where B is the amplitude of reflected SH-wave (this reflected wave is called regularly reflected wave), γ is the angle of incidence and

$$q = \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 + k^2 + \frac{M_{10}}{N_{10}}b_1^2 - \frac{\omega^2}{\beta_{h_1}^2}\right)}. \tag{14}$$

Similarly, the displacement in medium H_2 is given by

$$V^{refr} = \frac{1}{\sqrt{M_{20}p(x)r_2(z)}}D\,e^{-uz}\,e^{\iota\omega\left(t - \frac{x\sin\delta}{\beta_{h_2}}\right)}, \tag{15}$$

where D is the amplitude of refracted SH-wave (this refracted wave is called regularly refracted wave) and

$$u = \sqrt{\frac{N_{20}}{M_{20}}\left(\epsilon^2 + k^2 + \frac{M_{20}}{N_{20}}b_2^2 - \frac{\omega^2}{\beta_{h_2}^2}\right)}, \tag{16}$$

δ is the angle between the refracted wave and the z-axis, connected with the incidence angle γ by the Snell's law

$$\frac{\sin\gamma}{\beta_{h_1}} = \frac{\sin\delta}{\beta_{h_2}} = \frac{k}{\omega}. \tag{17}$$

Due to the corrugated interface, the nth order spectrum for the scattered reflected waves (these waves are called irregularly reflected waves) is given by (the arguments of p and r_i have been dropped)

$$V^{ir-refl} = \frac{1}{\sqrt{M_{10}p\,r_1}}\left[B_n e^{q_n z}\,e^{\iota\omega\left(t - \frac{x\sin\gamma_n}{\beta_{h_1}}\right)} + B_n' \, e^{q_n' z}\,e^{\iota\omega\left(t - \frac{x\sin\gamma_n'}{\beta_{h_1}}\right)}\right], \tag{18}$$

where B_n and B_n' are constants and

$$q_n = \sqrt{\frac{N_{10}}{M_{10}}\left(-\frac{\omega^2\cos^2\gamma_n}{\beta_{h_1}^2} + \epsilon^2 + \frac{M_{10}}{N_{10}}b_1^2\right)}, \tag{19}$$

$$q_n' = \sqrt{\frac{N_{10}}{M_{10}}\left(-\frac{\omega^2\cos^2\gamma_n'}{\beta_{h_1}^2} + \epsilon^2 + \frac{M_{10}}{N_{10}}b_1^2\right)}, \tag{20}$$

γ_n and γ_n' are the angles of the irregularly reflected waves with the z-axis.

Similarly, the nth order spectrum for the scattered refracted waves (these waves are called irregularly refracted waves) is given by

$$V^{ir-refr} = \frac{1}{\sqrt{M_{20}p\,r_2}}\left[D_n e^{-u_n z}\,e^{\iota\omega\left(t - \frac{x\sin\delta_n}{\beta_{h_2}}\right)} + D_n' \, e^{-u_n' z}\,e^{\iota\omega\left(t - \frac{x\sin\delta_n'}{\beta_{h_2}}\right)}\right], \tag{21}$$

where D_n and D'_n are constants and

$$u_n = \sqrt{\frac{N_{20}}{M_{20}}\left(-\frac{\omega^2 \cos^2 \delta_n}{\beta_{h_2}^2} + \epsilon^2 + \frac{M_{20}}{N_{20}}b_2^2\right)}, \quad (22)$$

$$u'_n = \sqrt{\frac{N_{20}}{M_{20}}\left(-\frac{\omega^2 \cos^2 \delta'_n}{\beta_{h_2}^2} + \epsilon^2 + \frac{M_{20}}{N_{20}}b_2^2\right)}, \quad (23)$$

δ_n and δ'_n are the angles of the irregularly refracted waves with the z-axis. The angles γ_n, γ'_n, δ_n and δ'_n are given by the following Spectrum theorem (Asano, 1960)

$$\sin \gamma_n - \sin \gamma = \frac{nk^*\beta_{h_1}}{\omega}, \qquad \sin \gamma'_n - \sin \gamma = -\frac{nk^*\beta_{h_1}}{\omega},$$

$$\sin \delta_n - \sin \delta = \frac{nk^*\beta_{h_2}}{\omega}, \qquad \sin \delta'_n - \sin \delta = -\frac{nk^*\beta_{h_2}}{\omega}. \quad (24)$$

Thus the total displacement V_1 in the medium H_1 is the sum of regularly reflected waves, irregularly reflected waves and incident wave:

$$V_1 = \frac{1}{\sqrt{M_{10}pr_1}}\left[e^{-qz} + Be^{qz} + \sum_{n=1}^{\infty} B_n e^{q_n z}e^{-\iota nk^*x}\right.$$
$$\left. + \sum_{n=1}^{\infty} B'_n e^{q'_n z}e^{\iota nk^*x}\right]e^{\iota\omega\left(t - \frac{x\sin\gamma}{\beta_{h_1}}\right)}. \quad (25)$$

Similarly, the total displacement V_2 in the medium H_2 is the sum of regularly refracted waves and irregularly refracted waves:

$$V_2 = \frac{1}{\sqrt{M_{20}pr_2}}\left[D e^{-uz} + \sum_{n=1}^{\infty} D_n e^{-u_n z}e^{-\iota nk^*x}\right.$$
$$\left. + \sum_{n=1}^{\infty} D'_n e^{-u'_n z}e^{\iota nk^*x}\right]e^{\iota\omega\left(t - \frac{x\sin\delta}{\beta_{h_2}}\right)}, \quad (26)$$

The constants B, D, B_n, D_n, B'_n and D'_n can be determined by satisfying the boundary conditions at the interface. In writing Eqs. (25) and (26), we have used Spectrum theorem.

3. Boundary Conditions

The boundary conditions to be satisfied at the corrugated interface $z = \zeta$ are the continuity of displacement and traction, that is

(I) $V_1 = V_2$,

(II) $M_1 \dfrac{\partial V_1}{\partial \nu} = M_2 \dfrac{\partial V_2}{\partial \nu}$,

where ν denotes the normal to the interface. Following Sokolnikoff (1956) and using relation (5), the boundary condition (II) can be written as

$$M_{10}pr_1(z)\left[\frac{\partial V_1}{\partial z} - \frac{\partial V_1}{\partial x}\zeta'\right]\frac{1}{\sqrt{1+\zeta'^2}}$$
$$= M_{20}pr_2(z)\left[\frac{\partial V_2}{\partial z} - \frac{\partial V_2}{\partial x}\zeta'\right]\frac{1}{\sqrt{1+\zeta'^2}}, \quad (27)$$

where ζ' is the derivative of ζ with respect to x. Insertion of Eqs. (25) and (26) in (I) and (II), and the use of the following substitutions

$$q = \iota Q, \qquad u = \iota U,$$
$$q_n = \iota Q_n, \qquad u_n = \iota U_n, \qquad q'_n = \iota Q'_n, \qquad u'_n = \iota U'_n, \quad (28)$$

where

$$Q = \sqrt{\frac{N_{10}}{M_{10}}\left(\frac{\omega^2}{\beta_{h_1}^2} - \epsilon^2 - k^2 - \frac{M_{10}}{N_{10}}b_1^2\right)},$$

$$U = \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\omega^2}{\beta_{h_2}^2} - \epsilon^2 - k^2 - \frac{M_{20}}{N_{20}}b_2^2\right)},$$

$$Q_n = \sqrt{\frac{N_{10}}{M_{10}}\left(\frac{\omega^2 \cos^2 \gamma_n}{\beta_{h_1}^2} - \epsilon^2 - \frac{M_{10}}{N_{10}}b_1^2\right)},$$

$$Q'_n = \sqrt{\frac{N_{10}}{M_{10}}\left(\frac{\omega^2 \cos^2 \gamma'_n}{\beta_{h_1}^2} - \epsilon^2 - \frac{M_{10}}{N_{10}}b_1^2\right)},$$

$$U_n = \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\omega^2 \cos^2 \delta_n}{\beta_{h_2}^2} - \epsilon^2 - \frac{M_{20}}{N_{20}}b_2^2\right)},$$

$$U'_n = \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\omega^2 \cos^2 \delta'_n}{\beta_{h_2}^2} - \epsilon^2 - \frac{M_{20}}{N_{20}}b_2^2\right)},$$

lead to:

$$\sqrt{r_2}\left[e^{-\iota Q\zeta} + B e^{\iota Q\zeta} + \sum_{n=1}^{\infty} B_n e^{\iota Q_n \zeta}e^{-\iota nk^*x}\right.$$
$$\left. + \sum_{n=1}^{\infty} B'_n e^{\iota Q'_n \zeta}e^{\iota nk^*x}\right]$$
$$= \sqrt{mr_1}\left[D e^{-\iota U\zeta} + \sum_{n=1}^{\infty} D_n e^{-\iota U_n \zeta}e^{-\iota nk^*x}\right.$$
$$\left. + \sum_{n=1}^{\infty} D'_n e^{-\iota U'_n \zeta}e^{\iota nk^*x}\right], \quad (29)$$

and

$$\sqrt{mr_1}\left[\left\{-Q + \frac{\iota r'_1}{2r_1} + \left(\frac{\omega \sin \gamma}{\beta_{h_1}} - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{-\iota Q\zeta}\right.$$
$$+ B\left\{Q + \frac{\iota r'_1}{2r_1} + \left(\frac{\omega \sin \gamma}{\beta_{h_1}} - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{\iota Q\zeta}$$
$$+ \sum_{n=1}^{\infty} B_n e^{-\iota nk^*x}\left\{Q_n + \frac{\iota r'_1}{r_1}\right.$$
$$\left. + \left(\frac{\omega \sin \gamma}{\beta_{h_1}} + nk^* - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{\iota Q_n\zeta}$$
$$+ \sum_{n=1}^{\infty} B'_n e^{\iota nk^*x}\left\{Q'_n + \frac{\iota r'_1}{r_1}\right.$$
$$\left.\left. + \left(\frac{\omega \sin \gamma}{\beta_{h_1}} - nk^* - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{\iota nk^*x}\right]$$
$$= \sqrt{r_2}\left[D\left\{-U + \frac{\iota r'_2}{2r_2} + \left(\frac{\omega \sin \delta}{\beta_{h_2}} - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{-\iota U\zeta}\right.$$
$$+ \sum_{n=1}^{\infty} D_n e^{-\iota nk^*x}\left\{-U_n + \frac{\iota r'_2}{2r_2}\right.$$
$$\left. + \left(\frac{\omega \sin \delta}{\beta_{h_2}} + nk^* - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{-\iota U_n\zeta}$$
$$+ \sum_{n=1}^{\infty} D'_n e^{\iota nk^*x}\left\{-U'_n + \frac{\iota r'_2}{r_2}\right.$$
$$\left.\left. + \left(\frac{\omega \sin \delta}{\beta_{h_2}} - nk^* - \frac{\iota p'}{2p}\right)\zeta'\right\}e^{-\iota U'_n\zeta}\right], \quad (30)$$

where $m = M_{10}/M_{20}$; r_i' and p' denote the derivatives of r_i and p with respect to z and x, respectively. From Eq. (13), with $B = 0$ (i.e. the incident wave), Eqs. (15), (26) and (17), the Snell's law; it is apparent that D, D_n and D_n' are connected with the transmission coefficients T, T_n and T_n' through the relation (cf. Singh et al., 1978)

$$\{T, T_n, T_n'\}$$
$$= \{De^{-u\zeta}, D_ne^{-u_n\zeta}, D_n'e^{-u_n'\zeta}\}e^{q\zeta}\sqrt{\frac{M_{10}r_1(\zeta)}{M_{20}r_2(\zeta)}}. \quad (31)$$

4. Solution of the First Order Approximation

For working out approximate solutions, we assume that the corrugation of the surface $z = \zeta$ is so small that higher powers of ζ may be neglected, so we have

$$e^{-\iota Q\zeta} = 1 - \iota Q\zeta. \quad (32)$$

We shall take

$$\{r_i(z),\ p(x)\} = \left\{e^{d_iz},\ e^{d_3x}\right\}, \quad i = 1,\ 2. \quad (33)$$

where d_1, d_2 and d_3 are constants. Substituting these into Eqs. (25) and (26), the first order approximation for B and D can be obtained by collecting the terms independent of x and ζ, as follows

$$(1 + B) = \sqrt{m}D, \quad (34)$$

$$\left(-Q + \frac{\iota d_1}{2}\right) + B\left(\frac{\iota d_1}{2} + Q\right) = \frac{D}{\sqrt{m}}\left(-U + \frac{\iota d_2}{2}\right). \quad (35)$$

These formulae give the values of B and D at the plane interface. B is known as reflection coefficient at the plane interface while D is connected to the transmission coefficient T at the plane interface, by the relation (31). Solving Eqs. (34) and (35), we obtain

$$B = \frac{2(-U + mQ) + \iota(d_2 - md_1)}{2(U + mQ) - \iota(d_2 - md_1)}, \quad (36)$$

$$D = \frac{4\sqrt{m}\ Q}{2(U + mQ) - \iota(d_2 - md_1)}. \quad (37)$$

In order to find the transmission coefficients T, we plug the values of r_i and D from (33) and (37) into (31), we obtain

$$T = \frac{4mQ\ e^{-(u-q)\zeta}\ e^{\sqrt{(d_1-d_2)\zeta}}}{2(U + mQ) - \iota(d_2 - md_1)}.$$

Approximating the exponential terms $e^{-(u-q)\zeta}$ and $e^{\sqrt{(d_1-d_2)\zeta}}$, similar to (32) and collecting the constant term (i.e. independent of x and ζ), the transmission coefficient T at the plane interface is obtained as

$$T = \frac{4mQ}{2(U + mQ) - \iota(d_2 - md_1)}. \quad (38)$$

Keeping in view the geometry of the problem, we see that expressions of B and D in (36) and (37) are same as obtained by Singh et al. (1978) for the relevant problem.

Next, for the solution of first order approximation for B_n and D_n, we collect the coefficients of $e^{-\iota nk^*x}$ and obtaining

$$B_n - \sqrt{m}D_n = \left[\sqrt{m}\left(\frac{d_1}{2} - \iota U\right)D\right.$$
$$\left. - \frac{d_2}{2}(1 + B) + \iota Q(1 - B)\right]\zeta_{-n}, \quad (39)$$

$$\left(\iota Q_n - \frac{d_1}{2}\right)B_n + \frac{1}{\sqrt{m}}\left(\frac{d_2}{2} + \iota U_n\right)D_n$$
$$= \left[\left\{Q^2 + \frac{d_1^2}{4} + \iota nk^*\left(\frac{d_3}{2} + \frac{\iota\omega\sin\gamma}{\beta_{h_1}}\right)\right\} \times (1 + B)\right.$$
$$\left. - \frac{1}{\sqrt{m}}\left\{U^2 + \frac{d_2^2}{4} + \iota nk^*\left(\frac{d_3}{2} + \frac{\iota\omega\sin\delta}{\beta_{h_2}}\right)\right\}D\right]\zeta_{-n}. \quad (40)$$

Similarly, equating the coefficients of $e^{\iota nk^*x}$, we obtain the first order approximation for B_n' and D_n' as follows

$$B_n' - \sqrt{m}D_n' = \left[\sqrt{m}\left(\frac{d_1}{2} - \iota U\right)D\right.$$
$$\left. + \iota Q(1 - B) - \frac{d_2}{2}(1 + B)\right]\zeta_n, \quad (41)$$

$$B_n'\left(\iota Q_n' - \frac{d_1}{2}\right) + \frac{1}{\sqrt{m}}\left(\frac{d_2}{2} + \iota U_n'\right)D_n'$$
$$= \left[\left\{Q^2 + \frac{d_1^2}{4} - \iota nk^*\left(\frac{d_3}{2} + \frac{\iota\omega\sin\gamma}{\beta_{h_1}}\right)\right\} \times (1 + B)\right.$$
$$\left. - \frac{1}{\sqrt{m}}\left\{U^2 + \frac{d_2^2}{4} - \iota nk^*\left(\frac{d_3}{2} + \frac{\iota\omega\sin\delta}{\beta_{h_2}}\right)\right\}D\right]\zeta_n. \quad (42)$$

The Eqs. (39)–(42) give the values of B_n, D_n, B_n' and D_n' as follows

$$B_n = \frac{\Delta_{B_n}}{\Delta_n}, \quad D_n = \frac{\Delta_{D_n}}{\Delta_n},$$
$$B_n' = \frac{\Delta_{B_n'}}{\Delta_n'}, \quad D_n' = \frac{\Delta_{D_n'}}{\Delta_n'}, \quad (43)$$

where

$$\Delta_{B_n} = \iota\zeta_{-n}\left[(1 + B)\left\{m\left(\frac{nk^*\omega\sin\gamma}{\beta_{h_1}} - \frac{d_1^2}{4} - Q^2\right) + \frac{d_2^2}{4}\right\}\right.$$
$$+ (1 - B)QU_n + D\sqrt{m}$$
$$\cdot\left(-UU_n - \frac{d_1d_2}{4} - \frac{nk^*\omega\sin\gamma}{\beta_{h_1}}\right.$$
$$\left. + \frac{d_2^2}{4} + U^2\right)$$
$$+ \iota\left\{(1 + B)\left(\frac{U_nd_2}{2} - \frac{md_3nk^*}{2}\right)\right.$$
$$- (1 - B)\frac{Qd_2}{2}$$
$$\left.\left. + D\sqrt{m}\left(\frac{Ud_2}{2} - \frac{U_nd_1}{2} + \frac{nk^*d_3}{2}\right)\right\}\right],$$

$$\Delta_{D_n} = \iota\zeta_{-n}\left[(1+B)\sqrt{m}\left(\frac{nk^*\omega\sin\gamma}{\beta_{h_1}} - \frac{d_1^2}{4}\right.\right.$$
$$\left.- Q^2 + \frac{d_1 d_2}{4}\right)$$
$$- \sqrt{m}(1-B)QQ_n$$
$$+ D\left(U^2 + \frac{d_2^2}{4}\right.$$
$$\left.- \frac{nk^*\omega\sin\gamma}{\beta_{h_1}} - \frac{md_1^2}{4} + mUQ_n\right)$$
$$+ \iota\left\{(1+B)\sqrt{m}\left(-\frac{Q_n d_2}{2} - \frac{d_3 nk^*}{2}\right)\right.$$
$$- \sqrt{m}(1-B)\frac{Qd_1}{2}$$
$$\left.\left.+ D\left(\frac{nk^*d_3}{2} + \frac{mUd_1}{2} + \frac{mQ_n d_1}{2}\right)\right\}\right],$$

$$\Delta_{B_n'} = \iota\zeta_n\left[(1+B)\left\{-m\left(\frac{nk^*\omega\sin\gamma}{\beta_{h_1}}\right.\right.\right.$$
$$\left.\left.+ \frac{d_1^2}{4} + Q^2\right) + \frac{d_2^2}{4}\right\}$$
$$+ (1-B)QU_n'$$
$$- \sqrt{m}D(UU_n' + \frac{d_1 d_2}{4}$$
$$\left.- \frac{nk^*\omega\sin\gamma}{\beta_{h_1}} - \frac{d_2^2}{4} - U^2\right)$$
$$+ \iota\left\{(1+B)\left(\frac{U_n'd_2}{2} + \frac{md_3 nk^*}{2}\right)\right.$$
$$- (1-B)\frac{Qd_2}{2}$$
$$\left.\left.- \sqrt{m}D\left(\frac{U_n'd_1}{2} - \frac{Ud_2}{2} + \frac{nk^*d_3}{2}\right)\right\}\right],$$

$$\Delta_{D_n'} = \iota\zeta_n\left[-(1+B)\sqrt{m}\left(\frac{nk^*\omega\sin\gamma}{\beta_{h_1}}\right.\right.$$
$$\left.+ \frac{d_1^2}{4} + Q^2 - \frac{d_1 d_2}{4}\right)$$
$$- \sqrt{m}(1-B)QQ_n'$$
$$+ D\left(U^2 + \frac{d_2^2}{4} + \frac{nk^*\omega\sin\gamma}{\beta_{h_1}}\right.$$
$$\left.- \frac{md_1^2}{4} + mUQ_n'\right)$$
$$+ \iota\left\{(1+B)\sqrt{m}\left(\frac{d_3 nk^*}{2} - \frac{Q_n'd_2}{2}\right)\right.$$
$$- \sqrt{m}(1-B)\frac{Qd_1}{2}$$
$$\left.\left.- D\left(\frac{nk^*d_3}{2} - \frac{mUd_1}{2} - \frac{mQ_n'd_1}{2}\right)\right\}\right],$$

$$\Delta_n = (U_n + mQ_n) + \frac{\iota}{2}(d_1 m - d_2),$$
$$\Delta_n' = (U_n' + mQ_n') + \frac{\iota}{2}(d_1 m - d_2).$$

The values of B and D appearing in the above expressions are given by (36) and (37). Here B_n and B_n' are reflection coefficients for the first order approximation of the corrugation, while the transmission coefficients for the first order approximation of the corrugation, the T_n and T_n' can be obtained from relation (31), on the similar way as T was obtained earlier, as

$$T_n = \sqrt{m}\, D_n, \qquad T_n' = \sqrt{m}\, D_n'. \qquad (44)$$

4.1 Special case

If we consider $\zeta_n = \zeta_{-n} = 0, (n \neq 1)$; $\zeta_1 = \zeta_{-1} = c/2$ then the boundary surface is given by $z = c\cos k^*x$, where c is the amplitude of the corrugation. From Eq. (43), we obtain the following formulae of B_1, D_1, B_1' and D_1' for the first order approximation of the corrugation as

$$B_1 = \frac{\Delta_{B_1}}{\Delta_1}, \qquad D_1 = \frac{\Delta_{D_1}}{\Delta_1},$$
$$B_1' = \frac{\Delta_{B_1'}}{\Delta_1'}, \qquad D_1' = \frac{\Delta_{D_1'}}{\Delta_1'}, \qquad (45)$$

where the values of Δ_{B_1}, Δ_{D_1}, $\Delta_{B_1'}$, $\Delta_{D_1'}$, Δ_1 and Δ_1' are given in Appendix A. The values of the transmission coefficients T_1 and T_1', in the present case, are given by

$$T_1 = \sqrt{m}\, D_1, \qquad T_1' = \sqrt{m}\, D_1'.$$

4.2 Particular cases

(a) When lateral and vertical heterogeneity of the media is removed then the medium H_1 and H_2 becomes transversely isotropic. Thus in this case, plugging $p(x) = r_1(x) = r_2(x) = 1$ into Eq. (8), we obtain $\epsilon = b_1 = b_2 = 0$ and hence the values of Q, U, Q_n, U_n, Q_n' and U_n' given in Eq. (28) through the Eqs. (14), (16), (19), (20), (22) and (23) reduce to

$$Q = \frac{\omega}{\beta_{h_1}}\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma,$$

$$U = \frac{\omega}{\beta_{h_1}}\sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)} = R,$$

$$Q_n = \frac{\omega}{\beta_{h_1}}\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma_n,$$

$$U_n = \frac{\omega}{\beta_{h_1}}\sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma_n\right)} = R_n,$$

$$Q_n' = \frac{\omega}{\beta_{h_1}}\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma_n',$$

$$U_n' = \frac{\omega}{\beta_{h_1}}\sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma_n'\right)} = R_n',$$
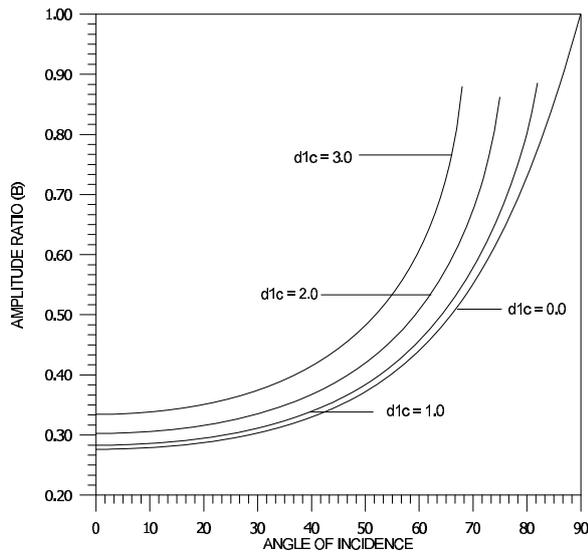
Fig. 2.   Variation of the Reflection coefficient at the plane interface (B) with the angle of incidence γ: when $d_2c = 0 = d_3c$ and $d_1c = 0.0, 1.0, 2.0, 3.0$.
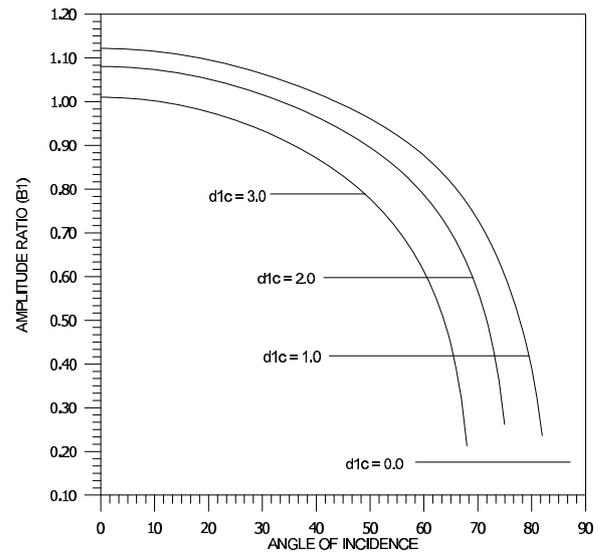
Fig. 4.   Variation of the Reflection coefficient for the first order approximation of corrugation (B_1) with the angle of incidence γ: when $d_2c = 0 = d_3c$ and $d_1c = 0.0, 1.0, 2.0, 3.0$.
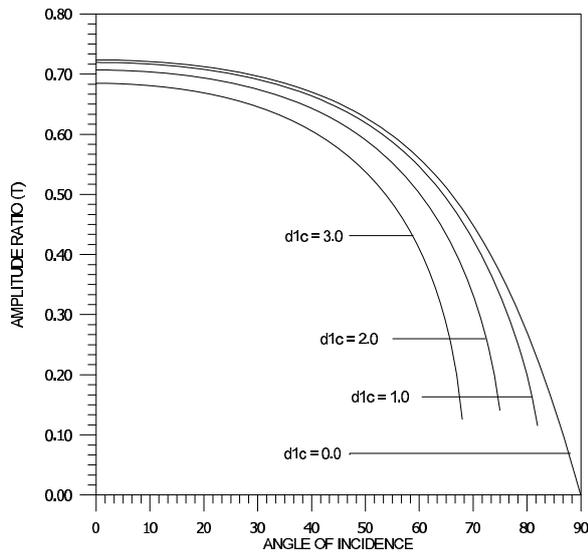
Fig. 3.   Variation of the Transmission coefficient at the plane interface (T) with the angle of incidence γ: when $d_2c = 0 = d_3c$ and $d_1c = 0.0, 1.0, 2.0, 3.0$.
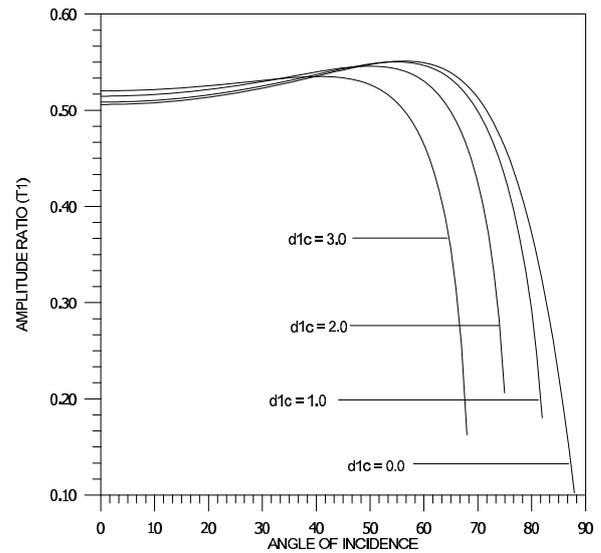
Fig. 5.   Variation of the Transmission coefficient for the first order approximation of corrugation (T_1) with the angle of incidence γ: when $d_2c = 0 = d_3c$ and $d_1c = 0.0, 1.0, 2.0, 3.0$.

With the help of these values, the coefficients given by Eqs. (36) and (37) for the plane interface, become

$$B = \frac{\frac{M_{10}}{M_{20}}\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma - \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)}}{\frac{M_{10}}{M_{20}}\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma + \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)}}, \qquad (46)$$

and

$$D = \frac{2\sqrt{\frac{N_{10}}{M_{10}}}\sqrt{\frac{M_{10}}{M_{20}}}\cos\gamma}{\frac{M_{10}}{M_{20}}\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma + \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)}}, \qquad (47)$$

and B_1, D_1, B_1' and D_1' are given in Appendix B. One can verify that using the relations, explained earlier

$$\{T, \ T_1, \ T_1'\} = \sqrt{m}\,\{D, \ D_1, \ D_1'\},$$

the reflection and transmission coefficients obtained in this case, coincide with those of Tomar and Saini (1997).

(b) When transverse isotropy of the media is removed, we shall be left with heterogeneity property in both the media. In this case, putting $N_{10} = M_{10}$ and $N_{20} = M_{20}$, in Eq. (28)
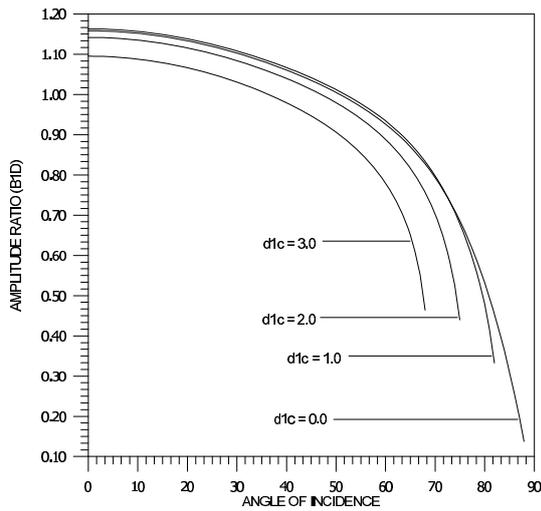
Fig. 6. Variation of the Reflection coefficient for the first order approximation of corrugation (B_1') with the angle of incidence γ: when $d_2c = 0 = d_3c$ and $d_1c = 0.0,\ 1.0,\ 2.0,\ 3.0$.
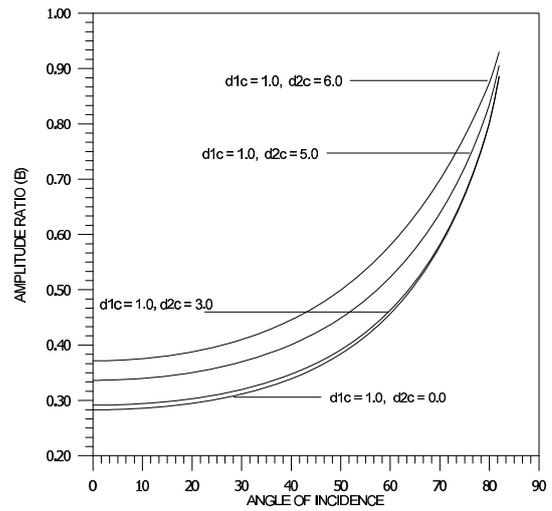
Fig. 8. Variations of the Reflection coefficient at the plane interface (B) with γ: when $d_3c = 0.0$, $d_1c = 1.0$ and $d_2c = 0.0,\ 3.0,\ 5.0,\ 6.0$.
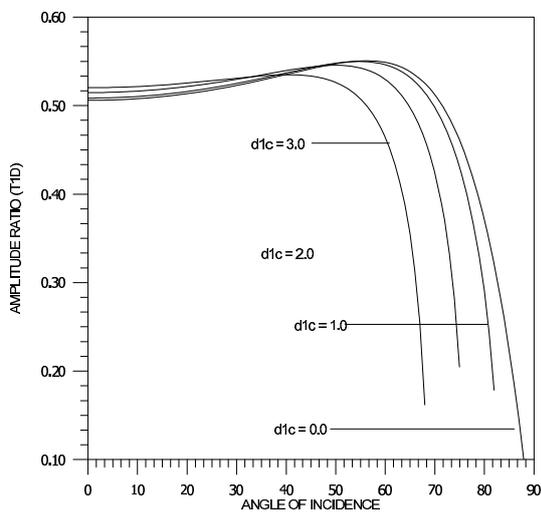
Fig. 7. Variation of the Transmission coefficient for the first order approximation of corrugation (T_1') with the angle of incidence γ: when $d_2c = 0 = d_3c$ and $d_1c = 0.0,\ 1.0,\ 2.0,\ 3.0$.
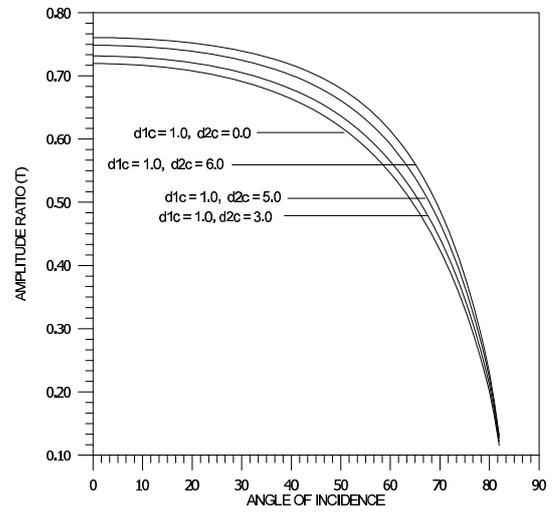
Fig. 9. Variations of the Transmission coefficient at the plane interface (T) with γ: when $d_3c = 0.0$, $d_1c = 1.0$ and $d_2c = 0.0,\ 3.0,\ 5.0,\ 6.0$.

with the Eqs. (14), (16), (19), (20), (22) and (23), we have

$$Q = -\iota\left(\epsilon^2 + b_1^2 - \frac{\omega^2\cos^2\gamma}{\beta_{h_1}^2}\right)^{1/2},$$

$$U = -\iota\left(\epsilon^2 + b_2^2 - \frac{\omega^2\cos^2\delta}{\beta_{h_2}^2}\right)^{1/2},$$

$$Q_n = -\iota\left(\epsilon^2 + b_1^2 - \frac{\omega^2\cos^2\gamma_n}{\beta_{h_1}^2}\right)^{1/2},$$

$$U_n = -\iota\left(\epsilon^2 + b_2^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2\delta_n\right)^{1/2},$$

$$Q_n' = -\iota\left(\epsilon^2 + b_1^2 - \frac{\omega^2\cos^2\gamma_n'}{\beta_{h_1}^2}\right)^{1/2},$$

$$U_n' = -\iota\left(\epsilon^2 + b_2^2 - \frac{\omega^2\cos^2\delta_n'}{\beta_{h_2}^2}\right)^{1/2}.$$

Using these modified values, the reflection and transmission coefficients for first order approximation can be obtained with the help of Eq. (43) and making changes accordingly in the Appendix-I. Also, it can be verified that in the present case, the boundary conditions (39)–(42) matches with those of Gupta (1987) for the corresponding problem.

(c) When only lateral heterogeneity of both media is removed, we are left with vertically heterogeneous and transversely isotropic media. In this case, $p(x) = 1$ so that $\epsilon = 0$. Putting these values in the Eq. (28) with Eqs. (14), (16), (19), (20), (22), (23), we get

$$Q = -\iota\sqrt{\frac{N_{10}}{M_{10}}\left(\frac{M_{10}}{N_{10}}b_1^2 - \frac{\omega^2}{\beta_{h_1}^2}\cos^2\gamma\right)},$$

$$U = \iota\sqrt{\frac{N_{20}}{M_{20}}\left(\frac{M_{20}}{N_{20}}b_2^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2\delta\right)},$$
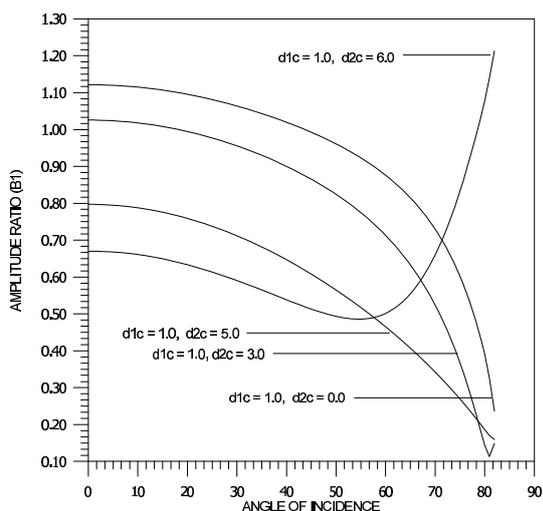
Fig. 10. Variations of the Reflection coefficient for the first order approximation of the corrugation (B_1) with γ: when $d_3c = 0.0$, $d_1c = 1.0$ and $d_2c = 0.0$, 3.0, 5.0, 6.0.
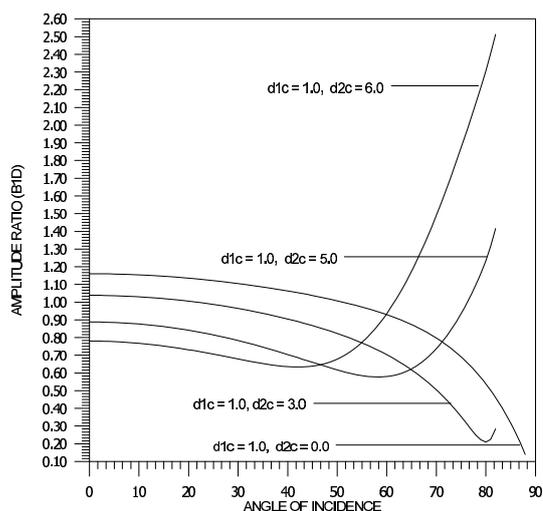
Fig. 12. Variations of the Reflection coefficient for the first order approximation of the corrugation (B_1') with γ: when $d_3c = 0.0$, $d_1c = 1.0$ and $d_2c = 0.0$, 3.0, 5.0, 6.0.
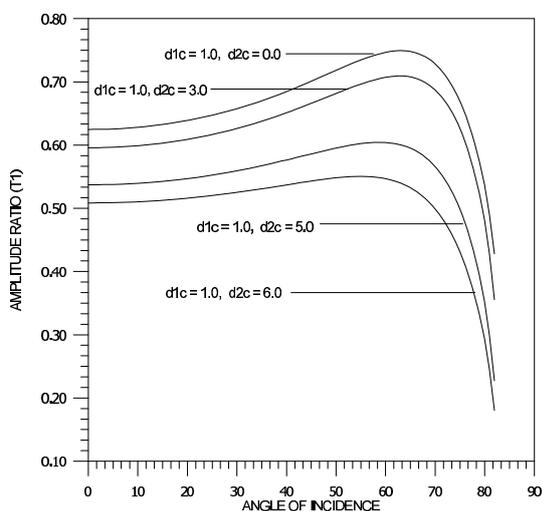
Fig. 11. Variations of the Transmission coefficient for the first order approximation of the corrugation (T_1) with γ: when $d_3c = 0.0$, $d_1c = 1.0$ and $d_2c = 0.0$, 3.0, 5.0, 6.0.
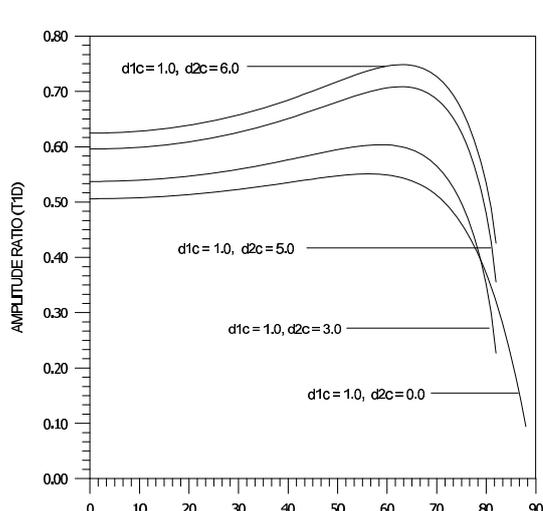
Fig. 13. Variations of the Transmission coefficient for the first order approximation of the corrugation (T_1') with γ: when $d_3c = 0.0$, $d_1c = 1.0$ and $d_2c = 0.0$, 3.0, 5.0, 6.0.

$$Q_n = -\iota \sqrt{\frac{N_{10}}{M_{10}}\left(\frac{M_{10}}{N_{10}}b_1^2 - \frac{\omega^2}{\beta_{h_1}^2}\cos^2\gamma_n\right)},$$

$$U_n = \iota \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{M_{20}}{N_{20}}b_2^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2\delta_n\right)},$$

$$Q_n' = -\iota \sqrt{\frac{N_{10}}{M_{10}}\left(\frac{M_{10}}{N_{10}}b_1^2 - \frac{\omega^2}{\beta_{h_1}^2}\cos^2\gamma_n'\right)},$$

$$U_n' = \iota \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{M_{20}}{N_{20}}b_2^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2\delta_n'\right)}.$$

With these modified values, the reflection and transmission coefficients for first order approximation are given by Eq. (43) with (44).

(d) When vertical heterogeneity is removed, we shall have laterally heterogeneous and transversely isotropic media. In

that case, $r_1(z) = r_2(z) = 1$ so that $b_1 = b_2 = 0$ and hence the values of Q, U, Q_n, U_n, Q_n' and U_n' given in Eq. (28) reduce to

$$Q = \iota \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 - \frac{\omega^2}{\beta_{h_1}^2}\cos^2\gamma\right)},$$

$$U = \iota \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2\delta\right)},$$

$$Q_n = \iota \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 - \frac{\omega^2}{\beta_{h_1}^2}\cos^2\gamma_n\right)},$$

$$U_n = \iota \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2\delta_n\right)},$$
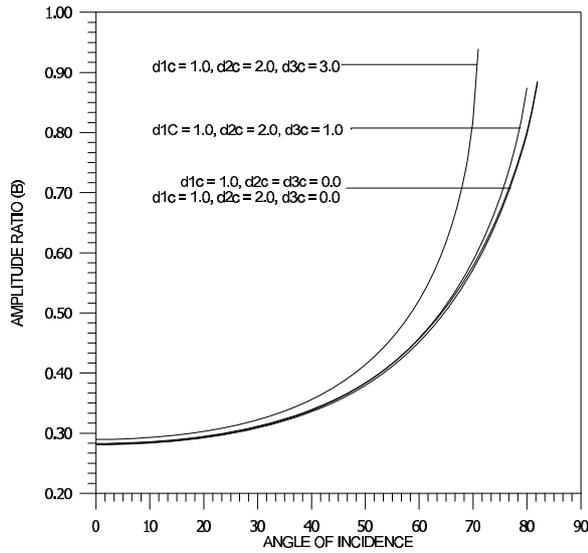
Fig. 14.   Variations of the Reflection coefficient at the plane interface (B) with γ: when $(d_1c,\ d_2c,\ d_3c) = (1.0,\ 0.0,\ 0.0)$; $(1.0, 2.0, 0.0)$; $(1.0, 2.0, 1.0)$; $(1.0, 2.0, 3.0)$.
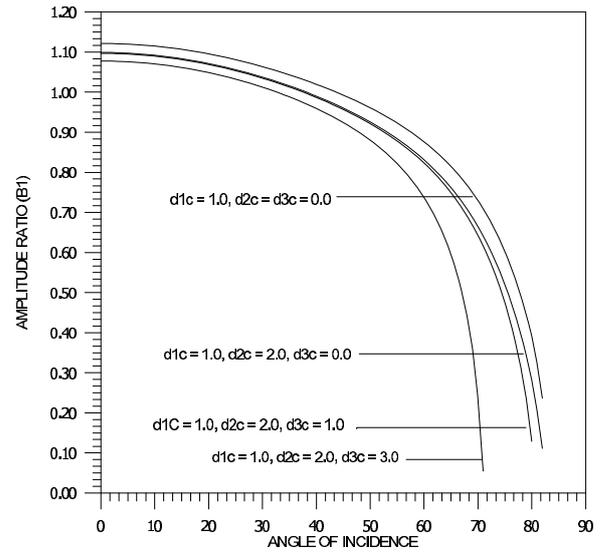
Fig. 16. Variations of Reflection coefficient for the first order approximation of the corrugation (B_1) with γ: when $(d_1c,\ d_2c,\ d_3c) = (1.0,\ 0.0,\ 0.0)$; $(1.0, 2.0, 0.0)$; $(1.0, 2.0, 1.0)$; $(1.0, 2.0, 3.0)$.
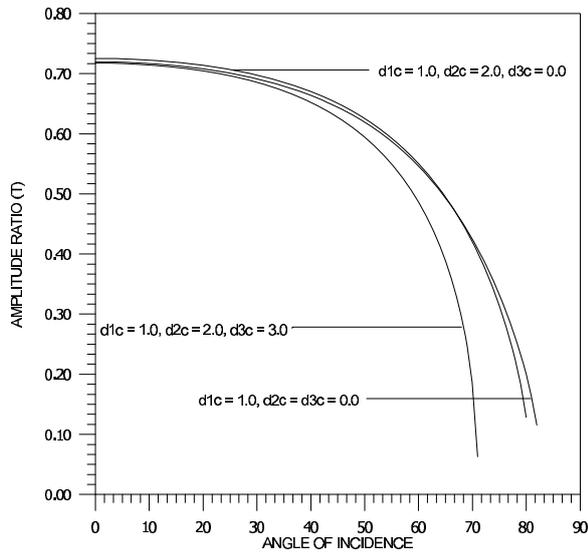
Fig. 15.   Variations of the Transmission coefficient at the plane interface (T) with γ: when $(d_1c,\ d_2c,\ d_3c) = (1.0,\ 0.0,\ 0.0)$; $(1.0, 2.0, 0.0)$; $(1.0, 2.0, 1.0)$; $(1.0, 2.0, 3.0)$.
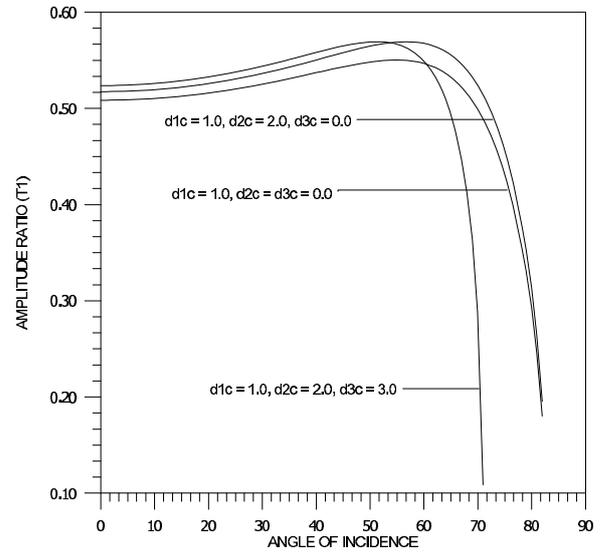
Fig. 17.   Variations of the Transmission coefficient for the first order approximation of the corrugation (T_1) with γ: when $(d_1c,\ d_2c,\ d_3c) = (1.0,\ 0.0,\ 0.0)$; $(1.0, 2.0, 0.0)$; $(1.0, 2.0, 1.0)$; $(1.0, 2.0, 3.0)$.

$$Q'_n = \iota \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 - \frac{\omega^2}{\beta_{h_1}^2}\cos^2 \gamma'_n\right)},$$

$$U'_n = \iota \sqrt{\frac{N_{10}}{M_{10}}\left(\epsilon^2 - \frac{\omega^2}{\beta_{h_2}^2}\cos^2 \delta'_n\right)}.$$

With these modified values, the reflection and transmission coefficients for first order approximation are given by the Eqs. (43) with (44).

(e)When both the heterogeneity and the transverse isotropy of the medium are removed, the medium H_1 and H_2 becomes homogeneous and isotropic elastic half spaces. In this case, by plugging $p(x) = r_1(x) = r_2(x) = 1$, $N_{10} = M_{10}$ and $N_{20} = M_{20}$ into Eq. (8), we get $\epsilon = 0$ and $b_1 = b_2 = 0$. And the values of Q, U, Q_n, U_n, Q'_n and U'_n,

given in the Eq. (28), reduce to

$$Q = \frac{\omega}{\beta_{h_1}}\cos\gamma, \qquad U = \frac{\omega}{\beta_{h_1}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)^{1/2},$$

$$Q_n = \frac{\omega}{\beta_{h_1}}\cos\gamma_n, \qquad U_n = \frac{\omega}{\beta_{h_1}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma_n\right)^{1/2},$$

$$Q'_n = \frac{\omega}{\beta_{h_1}}\cos\gamma'_n, \qquad U'_n = \frac{\omega}{\beta_{h_1}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma'_n\right)^{1/2}.$$

The values of the coefficients given by Eqs. (36) and (37) for the plane interface become

$$B = \frac{-U + mQ}{U + mQ}, \qquad D = \frac{2\sqrt{m}\,Q}{U + mQ}.$$
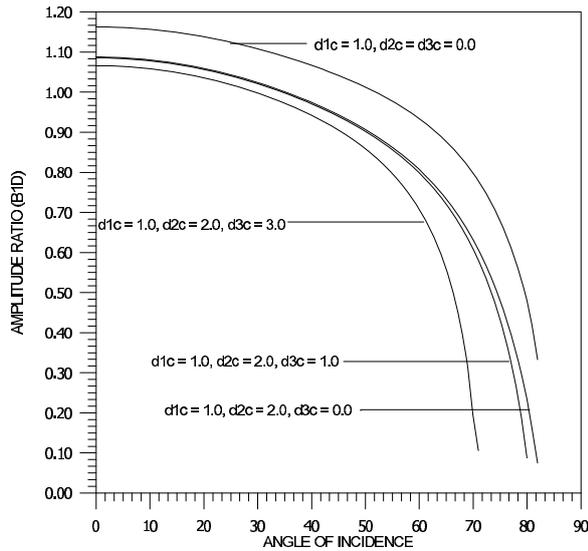
Fig. 18.   Variations of the Reflection coefficient for the first order approximation of the corrugation (B_1') with γ: when $(d_1c, \ d_2c, \ d_3c) = (1.0, \ 0.0, \ 0.0)$; $(1.0, \ 2.0, \ 0.0)$; $(1.0, \ 2.0, \ 1.0)$; $(1.0, \ 2.0, \ 3.0)$.

Fig. 20. Variations of Reflection and Transmission coefficients for the first order approximation of the corrugation (B_1) and (T_1) with k^*c: when $d_1c = 1.0, \ d_2c = 3.0, \ d_3c = 5.0$.

$$B_1' = \Delta\Bigg[-m(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + Q^2\right) + \sqrt{m}\,QQ_1(1-B)\Bigg],$$

$$+ (1-B)QU_1'$$

$$- D\sqrt{m}\left(UU_1' - U^2 - \frac{k^*\omega\sin\gamma)}{\beta_{h_1}}\right)\Bigg],$$

$$D_1' = \Delta\Bigg[-\sqrt{m}(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + Q^2\right)$$

$$+ D\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + U^2 + mUQ_1'\right)$$

$$- \sqrt{m}\,QQ_1'(1-B)\Bigg],$$

where

$$\Delta = \frac{\iota c}{2(U_1 + mQ_1)}.$$

These formulae give the reflection and refraction coefficients for the first order approximation of the corrugated interface between two uniform elastic half spaces. It can be verified that by removing the heterogeneity and anisotropy as explained above, the boundary conditions (29) and (30) match with those of Asano (1960).

5.   Numerical Results and Discussion

In order to study the effect of inhomogeneity and anisotropy numerically on reflection and transmission coefficients, when a plane SH-wave become incident obliquely at a corrugated interface between two half spaces H_1 and H_2, we have computed the modulus values of these coefficients for the model considered in section - Special case. For this purpose, we used the values of relevant parameters given as: For medium H_1: $N_{10} = 38.5 \times 10^{11} dyne/cm^2$, $M_{10} = 57.0 \times 10^{11} dyne/cm^2$, $\rho_1 = 2.2 g/cm^3$; For medium H_2: $N_{20} = 56.8 \times 10^{11} dyne/cm^2$, $M_{20} = 75.0 \times 10^{11} dyne/cm^2$,

Fig. 19.   Variations of the Transmission coefficient for the first order approximation of the corrugation (T_1') with γ: when $(d_1c, \ d_2c, \ d_3c) = (1.0, \ 0.0, \ 0.0)$; $(1.0, \ 2.0, \ 0.0)$; $(1.0, \ 2.0, \ 1.0)$; $(1.0, \ 2.0, \ 3.0)$.

The expressions of B_1, D_1, B_1' and D_1' in this case become

$$B_1 = \Delta\Bigg[m(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - Q^2\right)$$

$$+ (1-B)QU_1$$

$$+ D\sqrt{m}\left(-UU_1 + U^2 - \frac{k^*\omega\sin\gamma)}{\beta_{h_1}}\right)\Bigg],$$

$$D_1 = \Delta\Bigg[\sqrt{m}(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - Q^2\right)$$

$$- D\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - U^2 - mUQ_1\right)$$
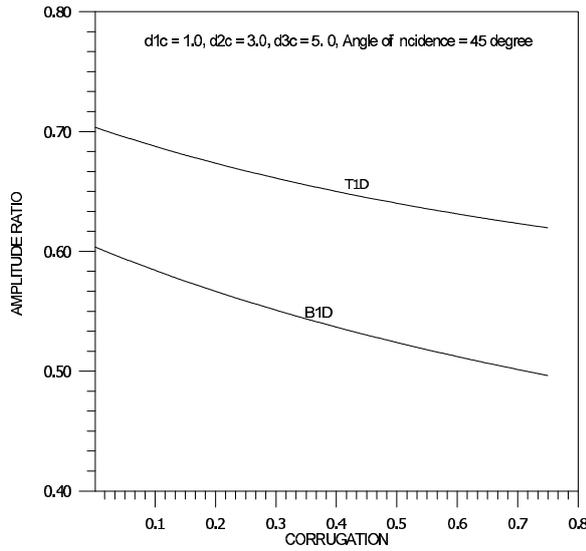
Fig. 21. Variations of the Reflection and Transmission coefficients for the first order approximation of the corrugation (B_1') and (T_1') with k^*c: when $d_1c = 1.0$, $d_2c = 3.0$, $d_3c = 5.0$ and $\gamma = 45^0$.
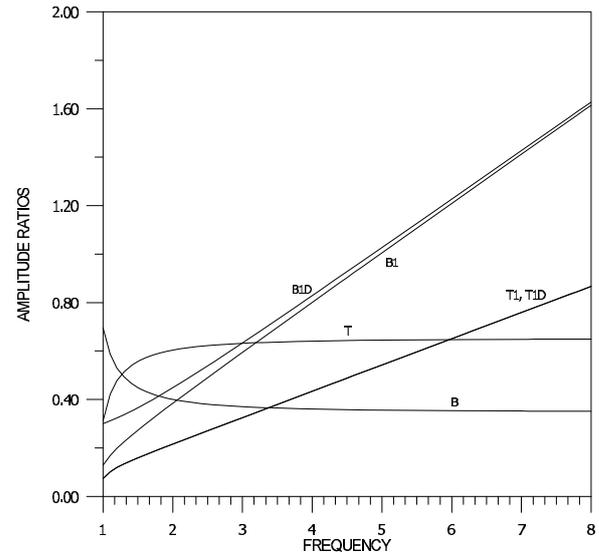
Fig. 22. Variations of the Reflection and Transmission coefficients for first order approximation of the corrugation (B_1), (T_1), (B_1') and (T_1') with frequency $(\omega c/\beta_{h_1})$: when $d_1c = 1.0$, $d_2c = 3.0$, $d_3c = 5.0$ and $\gamma = 45^0$.

$\rho_2 = 5.2 g/cm^3$; and $\omega c/\beta_{h_1} = 5.0$, c being the amplitude of corrugation and $k^*c = 0.00125$, wherever not mentioned. The non-dimensional quantities d_1c and d_2c represent the vertical heterogeneity factors in H_1 and H_2 respectively, while d_3c is lateral heterogeneity factor in both media.

(i)Effect of the vertical heterogeneity: Figures 2 and 3 show the variations of reflection and transmission coefficients with respect to angle of incidence at a plane interface between H_1 and H_2. Here we have taken variations in vertical heterogeneity in H_1 only, and H_2 a homogeneous one, that is, $d_2c = d_3c = 0$ and $d_1c = 0.0, 1.0, 2.0, 3.0$. The case $d_1c = d_2c = d_3c = 0$ represent that both the media are homogeneous. We observe from Figs. 2 and 3 that there is a significant effect of the vertical heterogeneity d_1c, on the reflection and transmission coefficients. It is found that the reflection coefficient B increases with d_1c, while the transmission coefficient T decreases with d_1c.

Figures 4–7 depict the variations of reflection and transmission coefficients with respect to angle of incidence for the first- order approximation of the corrugated interface. The effect of d_1c on these coefficients is clearly noticed. It is found that the values of reflection coefficients decreases with d_1c while in the transmission coefficient, initially there is a little increase with d_1c up to certain angle of incidence, beyond which it decreases with d_1c significantly. However, the critical angles are different for different non-zero values of d_1c.

Figures 8–13 show the effect of vertical heterogeneity in H_1 and H_2 (no lateral heterogeneity) on B, T, B_1, T_1, B_1' and T_1' respectively. The value of the vertical heterogeneity in medium H_1 is taken $d_1c = 1.0$, a constant while the vertically heterogeneity d_2c, in medium H_2 varies through values 0.0, 3.0, 5.0 and 6.0. From Figs. 8 and 9, we note that the values of B and T increase with d_2c. The effect of d_2c is maximum at $\gamma = 0°$ and minimum at $\gamma = 82°$, beyond which no reflection or transmission coefficient appear. In Fig. 10, the variation of reflection coefficient B_1 is

different for different value of γ and d_2c, while in Fig. 11, we note that the transmission coefficient T_1 decreases with d_2c. From Fig. 12, we note that when $d_2c = 0.0$, the coefficient B_1' decreases slowly with γ, but as d_2c takes values 3.0, 5.0 and 6.0 its value decreases up to certain angle and then start increases rapidly. In Fig. 13, we note that when $d_2c = 0.0, 3.0, 5.0$ and 6.0, the coefficient T_1' increases slowly up to certain value of γ and then decreases rapidly with γ.

Figures 14 and 15 show the combined effect of lateral and vertical heterogeneities on the reflection and transmission coefficients versus angle of incidence, at the plane interface between H_1 and H_2. We can notice from these figures that for different values of d_1c, d_2c and d_3c, the reflection coefficient B takes minimumum value at $\gamma = 0°$ which increases further with γ, while transmission coefficient T exhibit reverse behaviour.

Figures 16–19 show the effect of d_1c, d_2c and d_3c on B_1, T_1, B_1' and T_1'. We notice from these figures that the values of the coefficients B_1 and B_1' takes maximum value at $\gamma = 0°$ and then decreases with increase of γ, while the values of the coefficients T_1 and T_1' behaves alike. The values of later coefficients first increase slightly from the values attained at $\gamma = 0°$, then achieve maximum at certain angle of incidence and then decrease sharply. Also, the effect of d_3c is found more dominant beyond $\gamma = 50°$ angle of incidence.

(ii)Effect of corrugation: Firstly, it is to be noted that the coefficients B and T do not depend on the corrugation parameter k^*c, as they are the reflection and transmission coefficients respectively for the plane interface. To study the effect of k^*c on B_1, T_1, B_1' and T_1', we have computed them for very small values of k^*c. The reason is, because this method is not applicable for large values of corrugation. Here we have found that these coefficients are strongly effected by the corrugation k^*c. Figure 20 shows the variation of B_1 and T_1
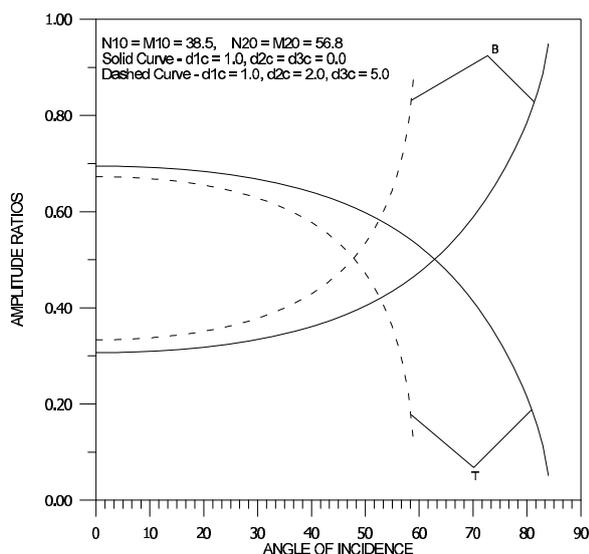
Fig. 23. Variations of the Reflection and Transmission coefficients at the plane interface (B) and (T) with γ: when $N_{10} = 38.5 = M_{10}$, $N_{20} = 56.8 = M_{20}$ and $d_1 c = 1.0$, $(d_2 c, d_3 c) = (0, 0)$; $(2.0, 5.0)$.
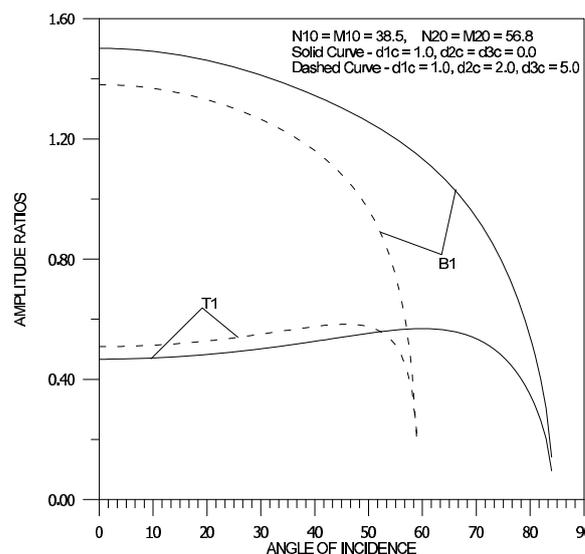
Fig. 24. Variations of the Reflection and Transmission coefficients for the first order approximation of the corrugation (B_1), (T_1), (B_1') and (T_1') with γ: when $N_{10} = 38.5 = M_{10}$, $N_{20} = 56.8 = M_{20}$ and $d_1 c = 1.0$, $(d_2 c, d_3 c) = (0, 0)$: $(2.0, 5.0)$.

Fig. 25. Variations of the Reflection and Transmission coefficients for the first order approximation of the corrugation (B_1), (T_1), (B_1') and (T_1') with γ: when $N_{10} = 38.5 = M_{10}$, $N_{20} = 56.8 = M_{20}$ and $d_1 c = 1.0$, $(d_2 c, d_3 c) = (0, 0)$; $(2.0, 5.0)$.

versus corrugation (k^*c) for $d_1 c = 1.0$, $d_2 c = 3.0$, $d_3 c = 5.0$, $\omega c / \beta_{h_1} = 5.0$ and $\gamma = 45°$. We notice that the values of B_1 and T_1 increase monotonically with the increase of k^*c. Figure 21 shows the variation of B_1' and T_1' versus k^*c with same values of parameters as taken for Fig. 20. We notice from this figure that the values of B_1' and T_1' decrease with k^*c. Also, we observe from these figures that the values of B_1 and T_1 are more than those of B_1' and T_1' as k^*c takes values through 0.0 to 0.8.

(iii) Effect of frequency: To study the effect of frequency of incident wave on reflection and transmission coefficients for both plane and corrugated interfaces, we fix the value of parameters as $d_1 c = 1.0$, $d_2 c = 3.0$, $d_3 c = 5.0$ and $\gamma = 45°$. Figure 22 shows that, these coefficients are influenced by the frequency $(\omega c / \beta_{h_1})$ of the incident SH-wave. The values of relection and transmission coefficients B and T decrease and increase respectively with $\omega c / \beta_{h_1}$ in the range $1.0 < \omega c / \beta_{h_1} < 1.6$ and thereafter they become constant for large values of $\omega c / \beta_{h_1}$. The coefficients for first order approximation of corrugation B_1, B_1', T_1 and T_1' increase with $(\omega c / \beta_{h_1})$.

(iv) Effect of heterogeneity and anisotropy: Figures 23 and 24 show the variation of B, T and B_1, T_1, B_1', T_1' with γ respectively, when both media are free from anisotropy and lateral heterogeneity, while vertical heterogeneity is present in H_1 only. From Fig. 23, we note that when anisotropy of both media are removed and the vertical heterogeneity in H_1 is fixed, the values of the reflection coefficient B takes minimum value at $\gamma = 0°$ and increases with γ, while the values of the transmission coefficient T possesses reverse behaviour. But, when the vertical heterogeneity is $d_2 c = 2.0$ and lateral heterogeneity is $d_3 c = 5.0$, the values of the coefficient B are greater and the values of T are less in comarision to the case of when $d_2 c = d_3 c = 0.0$. This shows that the lateral and vertical heterogeneities have significant effect on the reflection and transmission coefficients at plane interface.
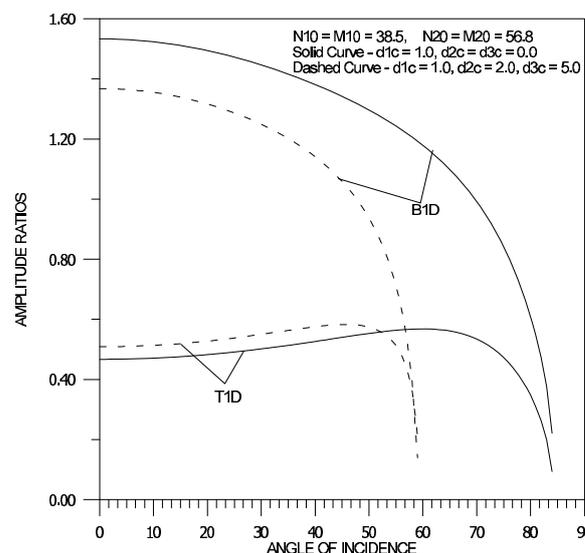
Figure 24 shows the variation of reflection and transmission coefficients for first order approximation of corrugation with γ, when anisotropy and lateral heterogeniety is removed and the vertical heterogeneity in H_1 and H_2 are taken as $d_1 c = 1.0$ and $d_2 c = 0.0$ respectively. We notice that the values of coefficient T_1 are less than the values of coefficient B_1. The same is true when the values $d_2 c = 2.0$ and $d_3 c = 5.0$ are introduced. A similar behaviour of coefficients B_1' and T_1' is observed from Fig. 25.

Figures 26–31, show the variation of B, T, B_1, T_1, B_1' and T_1' with γ for different values of the anisotropy factor in both media. These values are taken as $N_{10} / M_{10} = 1.0, 0.5, 0.7$ in H_1 and $N_{20} / M_{20} = 1.0, 0.8, 0.9$ in H_2, while the hetero-
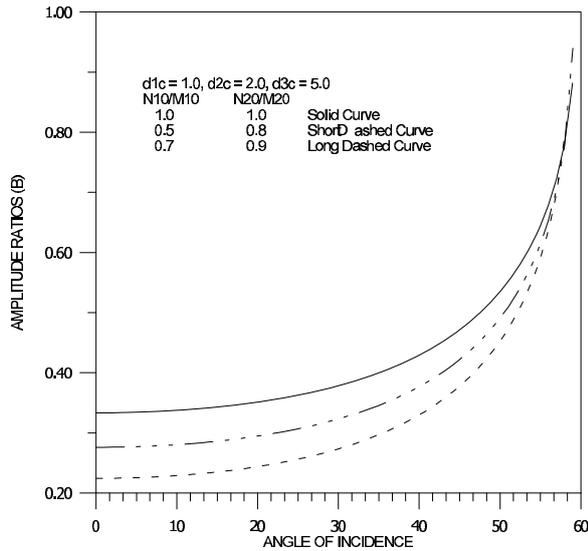
Fig. 26.   Variations of the Reflection coefficient at the plane interface (B) with γ: for $(N_{10}/M_{10},\ N_{20}/M_{20}) = (1.0,\ 1.0);\ (0.5,\ 0.8);\ (0.7,\ 0.9)$.
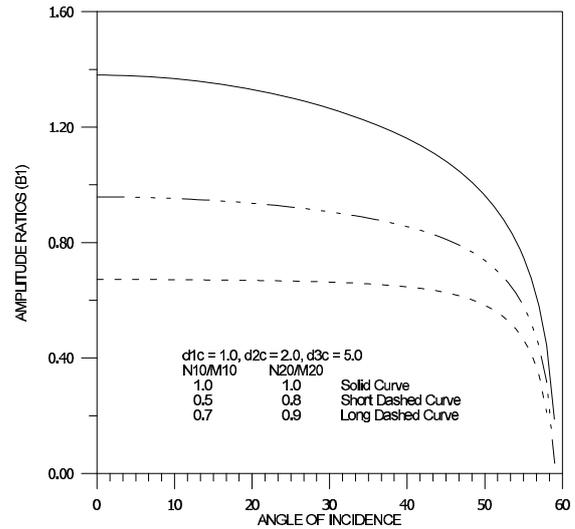
Fig. 28.   Variations of the Reflection coefficient for the first order approximation of the corrugation (B_1) with γ: when $(N_{10}/M_{10},\ N_{20}/M_{20}) = (1.0,\ 1.0);\ (0.5,\ 0.8);\ (0.7,\ 0.9)$.
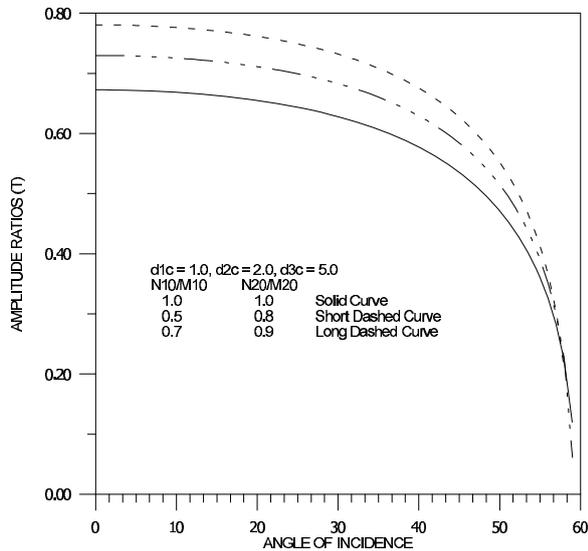
Fig. 27. Variations of the Transmission coefficient at the plane interface (T) with γ: for $(N_{10}/M_{10},\ N_{20}/M_{20}) = (1.0,\ 1.0);\ (0.5,\ 0.8);\ (0.7,\ 0.9)$.
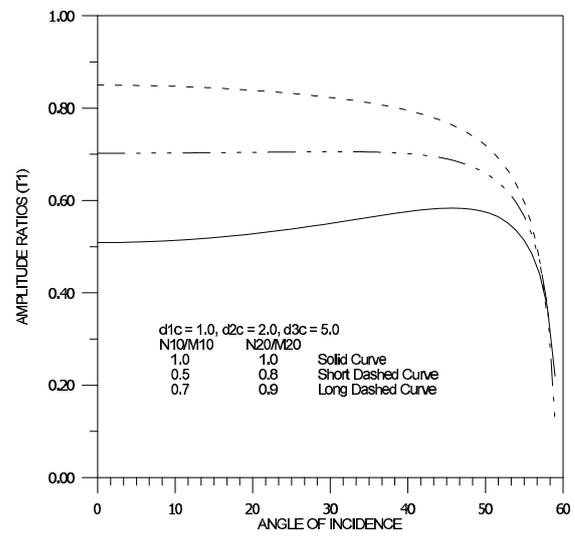
Fig. 29.   Variations of the Transmission coefficient for the first order approximation of the corrugation (T_1) with γ: when $(N_{10}/M_{10},\ N_{20}/M_{20}) = (1.0,\ 1.0);\ (0.5,\ 0.8);\ (0.7,\ 0.9)$.

geneities are taken as $d_1c = 1.0$, $d_2c = 2.0$, $d_3c = 5.0$. The case $N_{10}/M_{10} = N_{20}/M_{20} = 1.0$ correspond to isotropy of both the media. We notice from these figures that each coefficient, whether on the plane interface or on the corrugated interface, is significantly effected by the anisotropy of the media. The reflection and transmission coefficients for plane interface possess reverse behaviour while other coefficients behave alike.

6.   Conclusions

Using the Rayleigh's method, the formulae for reflection and transmission coefficients due to incident SH-wave at a corrugated interface between two laterally and vertically heterogeneous transversely isotropic elastic solid half spaces have been obtained in closed form. We conclude that

(1) Reflection and transmission coefficients for the first order approximation of the corrugation given by (43) with

(44) depend on ζ_{-n} and ζ_n i.e. on ζ, the corrugation of the interface. In the special case considered, if we remove the corrugation of the interface by putting $c = 0$ into the formulae given in Appendix-II, then $B_1 = T_1 = B_1' = T_1' = 0$ and we have only the non-vanishing expressions of B and T, given by (46) and (47), which are the reflection and transmission coefficients at the plane interface between two anisotropic homogeneous half-spaces.

(2) From the formulae (36)–(37) and (43) and our numerical results, we also conclude that the reflection and transmission coefficients strongly depend upon heterogeneity, anisotropy, frequency and angle γ of the incident SH-wave.

(i) Reflection and transmission coefficients for the first order approximation of the corrugation are found to be effected by the vertical heterogeneity factor d_1c. This effect is found least near normal incidence, when d_2c and d_3c are absent and
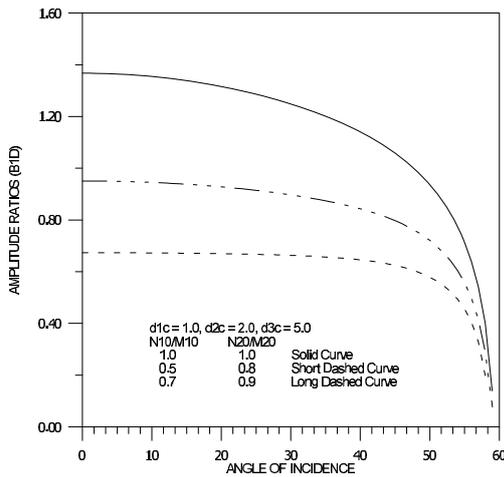
Fig. 30.   Variations of the Reflection coefficient for the first order approximation of the corrugation (B_1') with γ: when $(N_{10}/M_{10},\ N_{20}/M_{20}) = (1.0,\ 1.0);\ (0.5,\ 0.8);\ (0.7,\ 0.9)$.
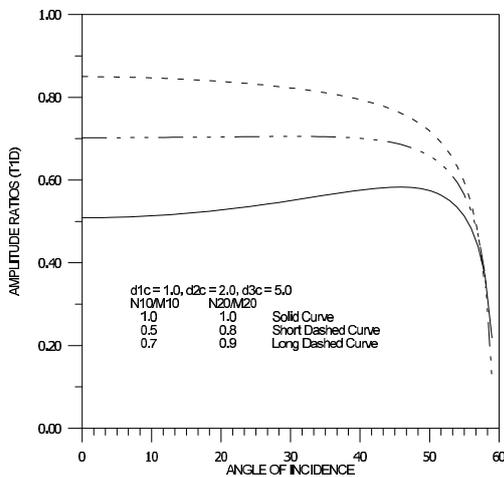
Fig. 31.   Variations of the Transmission coefficient for the first order approximation of the corrugation (T_1') with γ: when $(N_{10}/M_{10},\ N_{20}/M_{20}) = (1.0,\ 1.0);\ (0.5,\ 0.8);\ (0.7,\ 0.9)$.

k^*c is very small.

(ii) In the case when d_1c is fixed and d_2c is varied through the values 0.0, 3.0, 5.0 and 6.0 then the reflection and transmission coefficients are found to be influenced more by d_2c at $\gamma = 0°$.

(iii) As the corrugation increases, the values of B_1 and T_1 increase slowly while the values of B_1' and T_1' decrease slowly. The values of coefficients B and T remain uneffected.

(iv) When both the media are anisotropic and heterogeneous, these amplitude ratios are found to be influenced by frequency of the incident wave. For larger values of frequency, the values of the coefficients B and T remain almost constant.

(v) When lateral heterogeneity is removed and $d_1c = 1.0$, $d_2c = 0.0$, then the values of B increase with γ, while the values of T decrease with γ. On the other hand the remaining amplitude ratios decreases rapidly with γ. The introduction of lateral heterogeneity do not allow these coefficients to occur at and beyond $\gamma = 60°$.

(vi) The problems of Asano (1960), Gupta (1987) and Tomar and Saini (1997) have been obtained as a particular cases. Some new results have also been presented.

Appendix A.

$$\Delta_{B_1} = \frac{\iota c}{2}\left[(1+B)\left\{m\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - \frac{d_1^2}{4} - Q^2\right) + \frac{d_2^2}{4}\right\}\right.$$
$$+ (1-B)QU_1 + D\sqrt{m}\left(-UU_1 - \frac{d_1d_2}{4}\right.$$
$$\left. - \frac{k^*\omega\sin\gamma}{\beta_{h_1}} + \frac{d_2^2}{4} + U^2\right)$$
$$+ \iota\left\{(1+B)\left(\frac{U_1d_2}{2} - \frac{md_3k^*}{2}\right) - (1-B)\frac{Qd_2}{2}\right.$$
$$\left.\left. + D\sqrt{m}\left(\frac{Ud_2}{2} - \frac{U_1d_1}{2} + \frac{k^*d_3}{2}\right)\right\}\right],$$

$$\Delta_{D_1} = \frac{\iota c}{2}\left[(1+B)\sqrt{m}\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - \frac{d_1^2}{4} - Q^2 + \frac{d_1d_2}{4}\right)\right.$$
$$- \sqrt{m}(1-B)QQ_1$$
$$+ D\left(U^2 + \frac{d_2^2}{4} - \frac{k^*\omega\sin\gamma}{\beta_{h_1}} - \frac{md_1^2}{4} + mUQ_1\right)$$
$$+ \iota\left\{(1+B)\sqrt{m}\left(-\frac{Q_1d_2}{2} - \frac{d_3k^*}{2}\right)\right.$$
$$- \sqrt{m}(1-B)\frac{Qd_1}{2}$$
$$\left.\left. + D\left(\frac{k^*d_3}{2} + \frac{mUd_1}{2} + \frac{mQ_1d_1}{2}\right)\right\}\right],$$

$$\Delta_{B_1'} = \frac{\iota c}{2}\left[(1+B)\left\{-m\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + \frac{d_1^2}{4} + Q^2\right) + \frac{d_2^2}{4}\right\}\right.$$
$$+ (1-B)QU_1' - \sqrt{m}D$$
$$\cdot\left(UU_1' + \frac{d_1d_2}{4} - \frac{k^*\omega\sin\gamma}{\beta_{h_1}} - \frac{d_2^2}{4} - U^2\right)$$
$$+ \iota\left\{(1+B)\left(\frac{U_1'd_2}{2} + \frac{md_3k^*}{2}\right)\right.$$
$$- (1-B)\frac{Qd_2}{2}$$
$$\left.\left. - \sqrt{m}D\left(\frac{U_1'd_1}{2} - \frac{Ud_2}{2} + \frac{k^*d_3}{2}\right)\right\}\right],$$

$$\Delta_{D_1'} = \frac{\iota c}{2}\left[-(1+B)\sqrt{m}\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + \frac{d_1^2}{4} + Q^2 - \frac{d_1d_2}{4}\right)\right.$$
$$- \sqrt{m}(1-B)QQ_1'$$
$$+ D\left(U^2 + \frac{d_2^2}{4} + \frac{k^*\omega\sin\gamma}{\beta_{h_1}} - \frac{md_1^2}{4} + mUQ_1'\right)$$
$$+ \iota\left\{(1+B)\sqrt{m}\left(\frac{d_3k^*}{2} - \frac{Q_1'd_2}{2}\right)\right.$$
$$- \sqrt{m}(1-B)\frac{Qd_1}{2}$$
$$\left.\left. - D\left(\frac{k^*d_3}{2} - \frac{mUd_1}{2} - \frac{mQ_1'd_1}{2}\right)\right\}\right],$$

$$\Delta_1 = (U_1 + mQ_1) + \frac{\iota}{2}(d_1m - d_2),$$

$$\Delta_1' = (U_1' + mQ_1') + \frac{\iota}{2}(d_1m - d_2).$$

Appendix B.

$$B_1 = \frac{\iota c}{2\left(R_1 + \frac{M_{10}}{M_{20}}Q_1\right)}$$

$$\cdot \left[\frac{R_1 \omega}{\beta_{h_1}} \left\{ (1-B)\sqrt{\frac{N_{10}}{M_{10}}} \cos\gamma \right. \right.$$

$$\left. - D\sqrt{\frac{M_{10}}{M_{20}}} \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)} \right\}$$

$$+ \frac{M_{10}}{M_{20}}(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - Q^2\right)$$

$$\left. - D\sqrt{\frac{M_{10}}{M_{20}}}\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - R^2\right) \right],$$

$$D_1 = \frac{\iota c}{2\left(R_1 + \frac{M_{10}}{M_{20}}Q_1\right)}$$

$$\cdot \left[\sqrt{\frac{M_{10}}{M_{20}}}(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - Q^2\right) \right.$$

$$- D\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} - R^2\right)$$

$$+ \sqrt{\frac{M_{10}}{M_{20}}}\frac{Q_1\omega}{\beta_{h_1}}$$

$$\cdot \left\{ D\sqrt{\frac{M_{10}}{M_{20}}}\sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)} \right.$$

$$\left. \left. - (1-B)\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma \right\} \right],$$

$$B_1' = \frac{\iota c}{2\left(R_1' + \frac{M_{10}}{M_{20}}Q_1'\right)}$$

$$\cdot \left[\frac{R_1' \omega}{\beta_{h_1}} \left\{ (1-B)\sqrt{\frac{N_{10}}{M_{10}}} \cos\gamma \right. \right.$$

$$\left. - D\sqrt{\frac{M_{10}}{M_{20}}} \sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)} \right\}$$

$$- \frac{M_{10}}{M_{20}}(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + Q^2\right)$$

$$\left. + D\sqrt{\frac{M_{10}}{M_{20}}}\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + R^2\right) \right],$$

$$D_1' = \frac{\iota c}{2\left(R_1' + \frac{M_{10}}{M_{20}}Q_1'\right)}$$

$$\cdot \left[-\sqrt{\frac{M_{10}}{M_{20}}}(1+B)\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + Q^2\right) \right.$$

$$+ D\left(\frac{k^*\omega\sin\gamma}{\beta_{h_1}} + R^2\right)$$

$$- \sqrt{\frac{M_{10}}{M_{20}}}\frac{Q_1'\omega}{\beta_{h_1}}$$

$$\cdot \left\{ (1-B)\sqrt{\frac{N_{10}}{M_{10}}}\cos\gamma \right.$$

$$\left. \left. - D\sqrt{\frac{M_{10}}{M_{20}}}\sqrt{\frac{N_{20}}{M_{20}}\left(\frac{\beta_{h_1}^2}{\beta_{h_2}^2} - \sin^2\gamma\right)} \right\} \right].$$

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S. K. Tomar (e-mail: sktomar@yahoo.com) and J. Kaur