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The contribution of sprites to the global atmospheric electric circuit

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The global static electric field in the global atmospheric electric circuit resulting from mesospheric sprite discharges is inferred from a coupled model for the global static and dynamic electric fields derived from Maxwell's equations. It is found that the global atmospheric electric field from individual sprites is $\lesssim$ 44 mV/m, which can be measured with conventional ULF/ELF radio wave antennas at frequencies $\lesssim$ 4 Hz.

Key words: Sprites, lightning, global circuit.

1. Introduction

Sprites are transient luminous events in the mesosphere (Lyons, 1996; Sentman *et al.*, 1995; Boeck *et al.*, 1995; Franz *et al.*, 1990), which constitute a new element in the global atmospheric electric circuit (Su *et al.*, 2003; Sato and Fukunishi, 2003; Pasko *et al.*, 2002; Rycroft *et al.*, 2000). The impact of sprites on the global circuit has not yet been quantified, even though the quasi-static (DC) atmospheric electric field plays an important role in the global climate system (Carslaw *et al.*, 2002, and references therein). The global DC atmospheric electric field $\sim$ 150 V/m is mainly maintained by thunderstorm electric fields (Bering *et al.*, 1998; Hays and Roble, 1979; Roble and Hays, 1979). These electric fields exhibit a $\sim$20% diurnal variation with Universal Time, which is denoted the Carnegie curve (Füllekrug *et al.*, 1999; Holzer and Deal, 1956; Torreson *et al.*, 1946; Hoffmann, 1923). The contribution of sprites to the global DC atmospheric electric field may be similar to the contribution from particularly intense lightning discharges, $\sim$5–120 mV/m (Füllekrug, 2004). The main difficulty in measuring the contribution of sprites to the global DC atmospheric electric field is the inadequate sensitivity of ordinary electric field mills, $\gtrsim$ 1 V/m, such that other measurement technologies need to be considered. This study proposes a new methodology to infer the global DC atmospheric electric field of individual sprites from conventional global dynamic (AC) electric field measurements in the Ultra-Low and Extremely-Low Frequency (ULF/ELF) range made with radio wave antennas.

2. A Coupled Model for the Global DC and AC Electric Field

The global DC atmospheric electric field is derived from a solution of Maxwell's equations in a spherical geometry (Uman, 1974). For any charge Q in the atmosphere, specifically a sprite, the resulting global DC atmospheric electric field E_z points towards the centre of the Earth

$$E_z = \frac{Q}{4\pi\varepsilon_0 a^2}, \qquad (1)$$

where $a = 6371$ km is the Earth's equivolumetric radius and ε_0 is the electric permittivity. In this approach, the Earth and the ionosphere are considered to be concentric spherical shells. If a charge is deposited on the Earth's surface (e.g., by a lightning discharge) or delivered to the ionosphere (e.g., by a sprite), the potential difference between the Earth and the ionosphere $(V_e - V_i)$ changes and the electric field adjusts to the new charge configuration according to Eq. (1). The sprite charge is created instantaneously through quasi-static heating of the mesosphere by the causative lighting discharge. The charge is subsequently delivered to the ionosphere such that no mesospheric charge configuration prior to the sprite needs to be considered. The global AC electric field is derived from a solution of Maxwell's equations in a spherical geometry (Sentman, 1996, Bliokh *et al.*, 1980, p. 8–19), but it requires a conductivity model of the ionosphere (Füllekrug, 2005; Füllekrug *et al.*, 2002; Füllekrug, 2000; Sentman, 1990). The description of the global AC electric field with a weighted sum of spherical harmonic functions results in the short pulse approximation of the normal mode expansion with frequency dependent ionospheric heights

$$E_{AC}(\omega, \vartheta) = \frac{Ql}{4\pi\varepsilon_0 a^2 h_1(\omega)} \sum_{n=0}^{\infty} \frac{(2n+1)\,\omega P_n(\cos\vartheta)}{(\omega - \omega_n)(\omega + \omega_n^*)}. \qquad (2)$$

In this approach, the electric field spectrum $E_{AC}(\omega, \vartheta)$ is related to the intensity of the sprite, the geometric spreading of the radio wave and the ionospheric transfer function. The intensity of the sprite is characterised by the charge moment change Ql (Cummer and Füllekrug, 2001; Füllekrug *et al.*, 2001; Pasko *et al.*, 1998; Cummer *et al.*, 1998), which describes the amount of charge Q flowing within the body of a sprite of length $l/2$, and therefore includes the image current in the conductive ionosphere. The geometric spreading of the radio wave is described by the Legendre polynomials $P_n(\cos\vartheta)$ of degree n (an integer) at an angular

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distance ϑ from the sprite on a spheroidal Earth with radius a. The ionospheric transfer function is characterised by the frequency dependent conduction boundary $h_1(\omega) \approx 50$ km, where the displacement and conduction currents become equal, and the complex modal frequency

$$\omega_n = \sqrt{n(n+1)}\frac{c}{a}\sqrt{\frac{h_1(\omega)}{h_2(\omega)}}\left[1 - i\frac{\pi}{4}\left(\frac{s_1}{h_1(\omega)} + \frac{s_2}{h_2(\omega)}\right)\right] \tag{3}$$

(Füllekrug, 2000; Sentman, 1990; Greifinger and Greifinger, 1978), where $h_2(\omega) \approx 100$ km is the ionospheric height where the radio waves are reflected, $s_1 \approx s_2 \approx 2.5$ km are scale heights, which determine the exponential increase of the ionospheric conductivity in the atmosphere, ω_n^* is the complex conjugate of ω_n, and $c \approx 3 \cdot 10^8$ m/s is the speed of light.

The global AC electric field can be expressed in terms of the global DC atmospheric electric field

$$E_{AC}(\omega, \vartheta) = E_{DC}\frac{l}{h_1(\omega)}\sum_{n=0}^{\infty}\frac{(2n+1)\,\omega}{(\omega - \omega_n)(\omega + \omega_n^*)}P_n(\cos\vartheta) \tag{4}$$

by use of Eqs. (1) and (2). The uniform global AC electric field spectrum (Sentman, 1996, Eq. (38)) is calculated for a sprite with a charge moment change of $Ql = 1$ kC·km, e.g., a vertical charge transport of $Q = 20$ C in a sprite of $l/2 = 25$ km vertical extent (say 60–85 km), by integration along all source-receiver distances with the Gauss quadrature formula (Kautzleben, 1965, p. 21–24). The resulting electric field spectrum exhibits a surprising increase at frequencies $\lesssim 4$ Hz, which indicates a quasi-static component of the AC electric field (Fig. 1). This static term results from the Legendre polynomial P_0 of degree $n = 0$ in Eq. (4) ($P_0 \equiv 1$ for all source reciever distances ϑ), which corresponds to a constant electric field all around the globe. This peculiar property can readily be verified by calculating the global AC electric field spectrum without the Legendre polynomial P_0, i.e., extending the summation from $n = 1 \ldots \infty$ (Fig. 1). The resulting electric field spectrum now exhibits a decrease at frequencies $\lesssim 4$ Hz. It therefore seems possible to infer the global DC atmospheric electric field from an ultra-low frequency approximation of the global AC atmospheric electric field (Wait, 1962, p. 165). The straightforward analytic calculation of the asymptotic expansion of Eq. (2) for $\omega \to 0$ requires the treatment of the frequency dependence of the ionospheric heights $h_1(\omega)$ and $h_2(\omega)$ (Füllekrug, 2000) in the normal mode frequency ω_n (Eq. (3)), which is beyond the scope of this paper. Since neighbouring modal frequencies exhibit little interference with each other, we calculate the asymptotic expansion from the mode $n = 0$, where the Legendre polynomial P_0 and the complex modal frequency $\omega_0 = 0$ become the dominant terms such that Eq. (4) reduces to a basic scaling law

$$E_{DC} = \omega\,\frac{h_1(\omega)}{l}\,E_{AC}(\omega, \vartheta) \tag{5}$$

for ultra-low frequencies $\lesssim 4$ Hz, i.e., $\omega \ll \omega_1$. In this way, the models for the global AC and DC electric fields are coupled. The major scientific advance of the new AC/DC electric field model is that it is now possible to infer the global DC atmospheric field of sprites from conventional AC electric field measurements by use of the scaling law. Figure 1 illustrates the convergence of the approximated DC atmo-

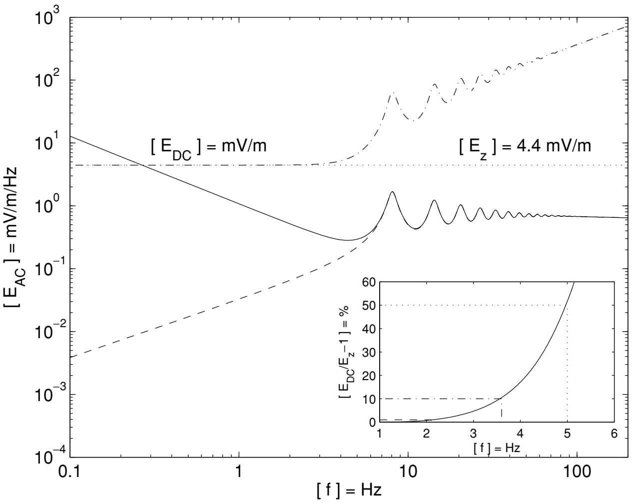

Fig. 1. The uniform AC electric field spectrum of a sprite discharge with a charge moment change of $Ql = 1$ kC·km exhibits an increase at frequencies $\lesssim 4$ Hz (solid line) which results from the ultra-low frequency approximation of Eq. (4) with the quasi-static $P_0 \equiv 1$ term. The uniform AC electric field spectrum calculated without the quasi-static P_0 term does not show this peculiar increase (dashed line). The approximated DC atmospheric electric field E_{DC} (Eq. (5)) converges at ultra-low frequencies $\lesssim 4$ Hz to a constant electric field value of $E_z = 4.4$ mV/m (dashed-dotted line), which is the exact value of the DC atmospheric electric field (dotted line) inferred from Eq. (1). The inset figure shows in more detail the frequency dependent deviation of the approximated DC atmospheric electric field value from the exact value (solid line) at frequencies of 2.1 Hz (dashed line), 3.6 Hz (dashed-dotted line) and 5.0 Hz (dotted line).

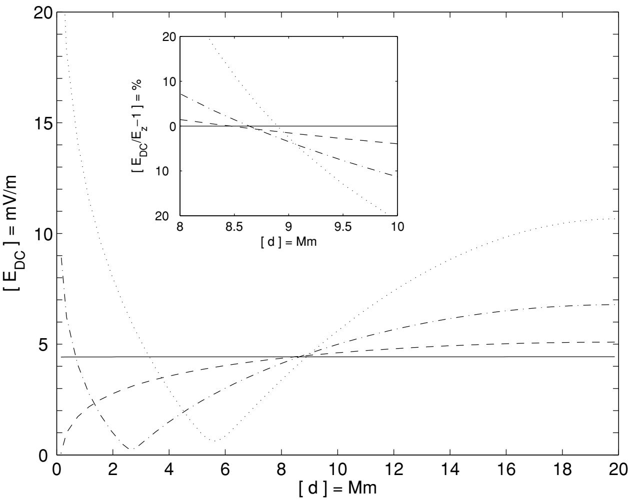

Fig. 2. The approximated DC atmospheric electric field E_{DC} of a sprite discharge with a charge moment change of $Ql = 1$ kC·km (Eq. (5)) exhibits a distinct source-receiver distance dependence at frequencies of 0.1 Hz (solid line), 2.1 Hz (dashed line), 3.6 Hz (dashed-dotted line) and 5.0 Hz (dotted line). The approximated DC atmospheric electric field is very close to the exact value $E_z = 4.4$ mV/m inferred from Eq. (1) at distances from 8–10 Mm. The inset figure shows in more detail the accuracy of the approximated DC atmospheric electric field relative to the exact value at frequencies of 0.1 Hz (solid line), 2.1 Hz (dashed line), 3.6 Hz (dashed-dotted line) and 5.0 Hz (dotted line).

spheric electric field E_{DC} (Eq. (5)) toward the exact value of the global DC atmospheric electric field of $E_z = 4.4$ mV/m (Eq. (1)) at ultra-low frequencies $\lesssim 4$ Hz for a sprite discharge with a charge moment change $Ql = 1$ kC·km. The ratio of the conduction boundary $h_1(\omega) \approx 50$ km and the vertical extent of the sprite $l/2 = 25$ km cancel such that the scaling law in Eq. (5) may be more roughly approximated with $E_{DC} \approx \omega \, E_{AC}(\omega, \vartheta)$.

3. The Accuracy of the Coupled AC/DC Electric Field Model

The accuracy of the coupled global AC/DC electric field model is frequency dependent (Fig. 1) as a result of the frequency dependent conduction boundary $h_1(\omega)$. The relative deviation of the approximated global DC atmospheric electric field (Eq. (5)) from the exact value (Eq. (1)) is $\sim$1% at 2.1 Hz, $\sim$10% at 3.6 Hz, and $\sim$50% at 5.0 Hz (Fig. 1, inset fig.). A deviation of 50% may seem to be large, but it is comparable to the uncertainty of the vertical extent of the sprite $l/2$, which exhibits a similar variability. In addition, the quoted accuracies are calculated from the uniform electric field spectrum, i.e., the integration of individual electric field spectra over all source-receiver distances. For an individual sprite event, the electric field spectrum needs to be calculated for one individual source-receiver distance. The source-receiver distance dependence of the approximated global DC atmospheric electric field is displayed in Fig. 2. At distances from 8–10 Mm, the approximated DC atmospheric electric fields are very close to the exact value of the DC atmospheric electric field $E_z = 4.4$ mV/m inferred from Eq. (1) for a vertical charge transport of $Q = 20$ C within a sprite of $l/2 = 25$ km vertical extent. The inset figure in Fig. 2 shows in more detail the relative error of the approximation which is $<$1% at 0.1 Hz for all source-receiver distances, $<$5% at 2.1 Hz from 8–10 Mm, $<$10% at 3.6 Hz from 8–10 Mm and $<$20% at 5.0 Hz from 8.5–9.5 Mm. It is evident from these results that the accuracy of the approximation decreases with increasing frequency (compare to Fig. 1, inset fig.). However, this effect can be compensated for by choosing a suitable location for the ULF/ELF radio wave antenna, at 8–10 Mm from the sprites.

4. Summary

The global DC atmospheric electric field of sprites can be determined from calibrated AC atmospheric electric field measurements at frequencies $\lesssim 4$ Hz with an error $\lesssim 10\%$ at source-receiver distances from 8–10 Mm. The largest charge moment changes observed on planet Earth are $\sim$ 10 kC·km (Füllekrug and Constable, 2000). This observation places an upper bound on the DC atmospheric electric field resulting from an individual sprite $\lesssim 44$ mV/m, or $\sim 3 \cdot 10^{-4} E_z$, where $E_z \approx 150$ V/m is the total global DC atmospheric electric field.

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