Initial studies of high latitude magnetic field data during different magnetospheric conditions

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We investigate the statistical properties of high-latitude magnetometer data for differing geomagnetic activity. This is achieved by characterizing changes in the nonlinear statistics of the geomagnetic field, by means of the Hurst exponent, measured from a single ground-based magnetometer station. The long-range statistical nature of the geomagnetic field at a local observation site can be described as a multifractional Brownian motion, thus suggesting the statistical structure required of mathematical models of magnetospheric activity. We also find that, in general, the average Hurst exponent for quiet magnetospheric intervals is smaller than that for more active intervals.

Key words: Storms, substorms, nonlinear geophysics, fractals.

1. Introduction

The solar wind continuously transports mass, energy and momentum into the magnetospheric cavity. This transfer is far from steady state even during solar minimum and rapid transfers of energy cause the magnetosphere to move from a relative low-energy state into more 'exited' states where the energy is suddenly dissipated in a series of global dynamical processes known as space storms. They are the most dramatic space weather phenomenon that significantly impact modern technology such as satellites, communication and power transmission systems. The development and morphology of space storms is well understood, but discussion is still open regarding exact causes magnetic storms (Gonzalez *et al.*, 1994). It is agreed that coronal mass ejections (CME), corotating streams and strong and prolonged values of the southward interplanetary magnetic field (IMF) are among the most important factors in the development of space storms (Tsurutani *et al.*, 1992; Richardson *et al.*, 2001; Huttunen *et al.*, 2003), but these factors alone are not sufficient nor necessary for the storm occurrence or development (Kamide *et al.*, 1998). Internal magnetospheric conditions, along with coupling and feedback between the ionosphere and magnetosphere, also play important roles in the initiation and development of storms (Daglis *et al.*, 2003).

Ground-based geomagnetic indices (Takalo *et al.*, 1999; Wanliss, 2004) and individual magnetometer stations (Vörös, 2000; Weigel *et al.*, 2002; Wanliss and Reynolds, 2003) have been used to provide excellent indicators of space weather conditions. Part of the reason for this is the property of the earth's magnetic field lines to focus and converge as they approach the earth. These field lines extend

far into space and since they are connected to the earth, nonlinear plasma processes that occur far away are mapped all the way down to the earth. Observation of ground-based magnetometer stations can thus serve as a remote sensing tool of distant magnetospheric features and processes.

Over the years, several indices were developed to monitor geomagnetic activity. The most commonly used are the disturbance storm time index (DST), the planetary index (Kp) and the auroral electrojet index (AE, AU and AL). These indices provide global information about current magnetospheric activity based on different inputs at different locations around the globe. For example, the Dst index is thought to measure the strength of the equatorial ring current that causes a decrease in the horizontal component of the magnetic field. It is derived hourly from data collected at four magnetic observatories: Hermanus, Kakioka, Honolulu, and San Juan. These observatories are located at middle and low latitudes around the globe away from the influences caused by the auroral and equatorial electrojets; the Kp index is calculated from observatories at high latitudes. For this reason the output of these indices will give us an idea of the geomagnetic activity from a global perspective. But if we are interested in the local aspects of geomagnetic activity, i.e. to forecast the geomagnetic conditions for Hydro-Quebec or other power utilities, we need to develop ways to understand the geomagnetic activity in a more localized way. This is especially important since temporal fluctuations of the geomagnetic field depend on geographic location and time (Weigel *et al.*, 2002).

In this paper we extend previous analyses that considered only global statistics to study the differences between quiet and active magnetospheric times (Wanliss, 2004, 2005), and which were used to suggest the possibility of a phase transition at space storm onset (Wanliss and Dobias, 2006). But global studies only give average behavior rather than local information. Our goal is to learn about local behavior of
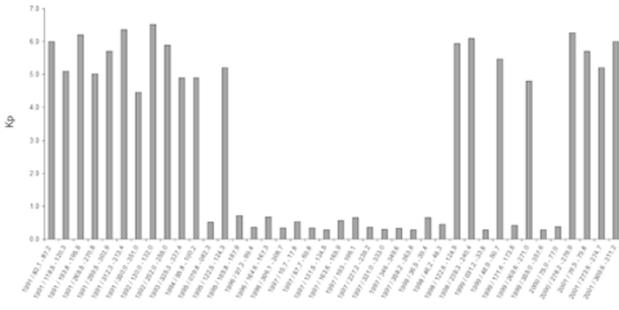
Fig. 1. Mean Kp values for the events analyzed from 1991 to 2001 are shown chronologically. The labels on the horizontal axis indicate the year and the start day through the end day using day of year notation.
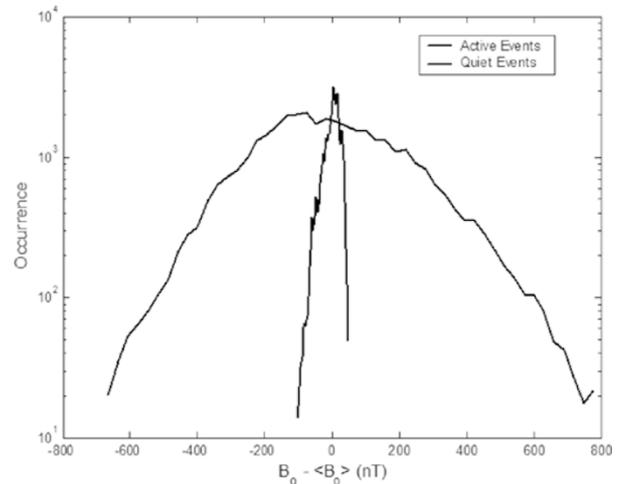

Fig. 2. Distribution functions for the active and quiet events analyzed. The solid line represents the averaged distributions for the 20 quiet events and the dash-dot line represents the averaged distributions for the 20 active events.

the magnetic field, for differing geomagnetic activity. We will characterize changes in the nonlinear statistics of the Earth's magnetic field, by means of the Hurst exponent, measured from a single ground-based magnetometer station. The changes in statistics can be used as a local indicator of the magnetospheric conditions, which may be useful to develop reliable warning and forecasting systems using information not available in geomagnetic indices.

A second objective is to determine the long-range statistical nature of the geomagnetic field at a local observation site. If the time series can be described as a particular statistical process—Brownian motion for example—then this knowledge can be used for future space weather modeling purposes. The statistical structure of the magnetometer time series will provide key requirements for the development of mathematical models. MHD models, for example, should correctly reproduce the correct statistical structure that we observe in the data.

The work performed and the results found are presented in the following pages. Section 2 describes the criteria used to select the data. In Section 3 we briefly describe the analysis techniques employed and the results of the different tests are presented in Section 4. Finally Section 5 discusses the results and compares them with those obtained by different authors.

2. Data Selection

We chose the three hour Kp index to discriminate between different levels of magnetospheric activity. We could have used other indices, for example DST or AE, but we chose Kp since we considered that it, as a mid- to high-latitude index, would best reflect the mean magnetospheric activity. Several methods for the classification of geomagnetic activity using the Kp index have been proposed and used by different authors. Gosling *et al*. (1991) used Kp to classify several levels of geomagnetic activity in more detailed fashion ranging from small storms to major storms. Bartels (1963) used the criteria for selecting quiet and active events based on Kp≤1 as an indicator of quiet periods, and Kp≥4 as an indicator of disturbed periods (Bartels, 1963; Rangarajan and Iyemori, 1997). In this work our interest focuses on two averaged geomagnetic states: active and quiet, so we adopted the Bartels criteria.

Data selected for quiet times (QT) were based on those periods between 1991 and 2001 where Kp≤1 for not less

than two days. The average length of the quiet events selected was 2.6 days. On the other hand, active events were selected from those periods of time having a Kp≥4 for no less than a day. Twenty active events matching these criteria were selected with an average length of 2.2 days. The length of each event is determined only by continuous intervals where the Kp matches the criterion. Once the Kp value moves outside the criterion, it sets the boundaries to that particular event. Figure 1 shows the mean Kp values of all the selected events, active and quiet, chronologically from 1991 to 2001. Most of the active events are close to solar maxima while the majority of the quiet events occur near solar minimum (1997).

We selected 40 events with the given criteria using the CANOPUS Fort Churchill magnetometer station (FCHU) as the primary data source. These data have a cadence of 5 (five) seconds. The reason behind the selection of this source is its geographic location (58.76N and 265.91W), which is frequently in the auroral oval. This location has the particular advantage that the data for the selected events are consistent with the location of stations used to make the Kp index, thus the Kp selection criteria would be more likely to accurately discriminate activity levels in this study.

The distribution functions of the magnetic field for QTs and ATs are presented in Fig. 2. In this plot the wider curve characterizes the magnetic field behavior during AT, while the thin curve represents the QT. The different nature of these two curves suggests the existence of different statistical processes dominating both data sets. The AT distribution is Gaussian in nature, but for QTs the trend is more asymmetrical and skewed to negative values.

3. Analysis Technique

We began analysis of other statistical properties by investigating scaling properties. The presence of possible scaling can be initially estimated by implementation of a power spectral density (PSD) analysis. The slope β obtained from the plot of PSD vs. frequency determines the correlation level of the signal. If $P(\nu) \propto \nu^{-\beta}$, and $-1 < \beta < 1$
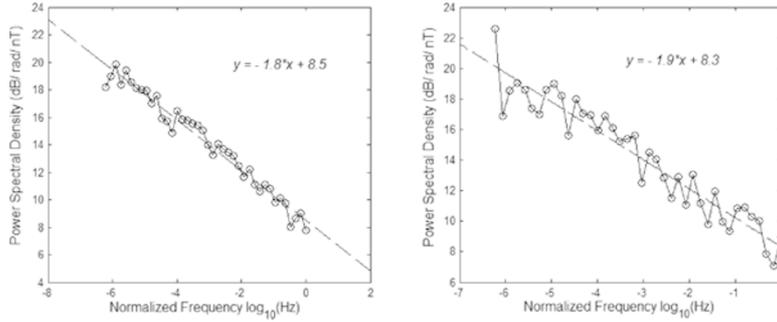
Fig. 3. Power spectral density for the quiet event of 1999, day of year 31 with $\beta = 1.8$ (left) and active event of 1995, day of year 122 with $\beta = 1.9$ (right). The mean Kp values for these QT and AT were 0.3 and 5.2 respectively.

then the signal is fractional Gaussian noise (fGn) and it is stationary, which means that the signal is statistically invariant by translation in time. A signal $B(t)$ that displays fractional Brownian motion (fBm) is one for which both the real and imaginary components of the Fourier amplitudes $\overline{\psi(\nu)} = 0$ are Gaussian-distributed random variables (Hergarten, 2002). As well, the mean of the Fourier amplitudes and $\overline{\psi(\nu)\psi(\nu')} = P(\nu)\delta(\nu - \nu')$. This means that for $\beta = 2$, fBm reduces to the random walk with power law spectrum varying as an inverse square. Signals with scaling exponents above $\beta = 2$ are called persistent, because if the data at some point have $B(t_{i+1}) > B(t_i)$, for example, then the probability is greater than 0.5 that $B(t_{i+2}) > B(t_{i+1})$. Signals with exponents below 2 are called antipersistent because if $B(t_{i+1}) > B(t_i)$, the probability is greater than 0.5 that $B(t_{i+2}) < B(t_{i+1})$. Typically, fBm is nonstationary, and detection of memory is a delicate task. Nonstationarity means that statistical properties are not constant through the signal, and traditional analysis methods that assume stationarity cannot be used.

If the signal is fractional fBm, it exhibits power-law scaling with slope $1 < \beta < 3$. In this case the signal is nonstationary but has stationary increments over a range of scales. For fBm

$$\beta = 2H + 1,$$

where H is the scaling exponent also known as the Hurst exponent. The special case where $\beta = 2(H = 0.5)$ indicates Brownian motion. Figure 3 displays the PSD vs. frequency for a QT (a) and AT (b) for which the average Kp was 0.3 and 5.2, respectively. The slope (indicates that time series falls within a fBm process suggesting that nonstationary processes are dominant during the time scale of the event.

The problem with spectral analysis, certainly in the context of the present study, is that it assumes a stationary signal. When it is implemented with nonstationary signals, spectral analysis is incapable of distinguishing frequency content hidden by the presence of nth order polynomial trends and requires long time-series for reasonable accuracy (Stanley *et al.*, 1999). To determine the self similarity parameter H we implement detrended fluctuation analysis (DFA) developed by Peng *et al.* (1995) and recently implemented in space physics research by Wanliss (2004, 2005). The technique is designed to determine the scaling exponent of nonstationary signal and provides better precision

than the power spectral analysis and other classical techniques.

In DFA the time average of the time series is subtracted from the original series and then it is integrated. Once the series is integrated, it is divided into boxes of equal size n. In each box a linear least squares line is fit to the data, representing the trend of the series in that particular box. The next step is to remove the local trend in each box. The characteristic size of the fluctuations $F(n)$, is then calculated as the root mean squared deviation between the signal and its trend in each box. The process is repeated over all time scales (box sizes). The presence of scaling is indicated by a power-law relationship between $F(n)$ and n as follows:

$$F(n) \propto n^{\alpha},$$

where $\alpha = H$, within the set (0,1), is the scaling exponent. The slope of a log-log curve of $F(n)$ vs. n indicates the value of the scaling exponent. If $\alpha = 0.5$ then the signal is white noise. A value of $\alpha < 0.5$ indicates that the data is uncorrelated (antipersistent) and if $0.5 < \alpha < 1$ then there is correlation in the time series (i.e., long term memory).

We analyzed the QT and AT time series from two different approaches. The first approach measures long term correlation over the *entire* event, and a single value of H is obtained for the entire series; we call this a monofractal approach since it assumes the presence of only a single scaling exponent. Next, we extended this approach to the time-dependent case where the scaling exponent is calculated in patches along the series. Rather than simply probe the existence of correlated behaviour over the entire time series, what we do is find a "local measurement" of the degree of long-range correlations described by the time variations of the scaling exponent. The probe used is the observation box of length 10000 data points; this box is placed at the beginning of the data, and then the scaling exponent was calculated for the data contained in the box. Experiments with synthetic data demonstrated that reasonable accuracy could be achieved for series of this length. Next, the box was shifted one point along the series, and the scaling exponent for the new box was calculated. This procedure was iterated for the entire sequence. This time-dependent approach allows one to consider a time series dominated by multi-scale processes or multifractional Brownian motion (mfBm). The mfBm is a generalised version of fBm in which the scaling exponent is no longer a constant, but a function of the time
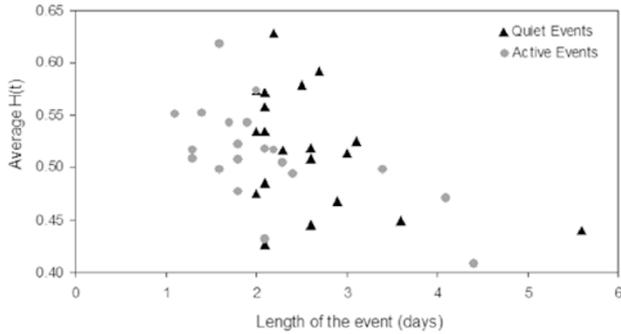
Fig. 4. Distributions of the Hurst exponent for quiet and active events vs. the event length. A single exponent was calculated for each event. No direct evidence was found to suggest that the Hurst exponent is affected by the event's length. The average Hurst exponent for quiet times is $\langle H_{QT} \rangle = 0.52 \pm 0.06$ and for active times $\langle H_{AT} \rangle = 0.51 \pm 0.05$. The difference is statistically insignificant.
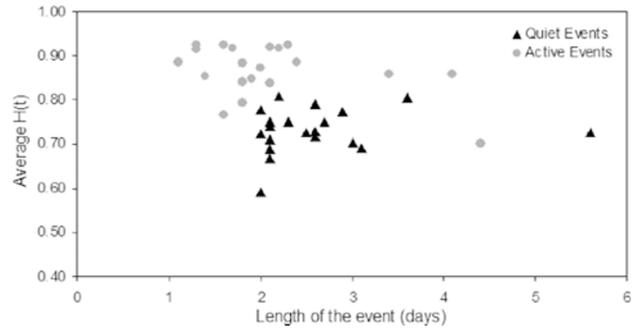

Fig. 5. Distributions of the time average Hurst coefficients for quiet and active events vs. the event length. No direct dependences were found that the average Hurst exponent is affected by the duration of the event. The average Hurst exponent for quiet times is $\langle H_{QT} \rangle = 0.73 \pm 0.05$ and for active times $\langle H_{AT} \rangle = 0.87 \pm 0.06$.

index (Peltier and Levy Vehel, 1995). In this case the increments of mfBm are nonstationary and the process is no longer self-similar.

4. Results

4.1 Time-independent analysis

Figure 4 summarizes the values obtained for H from the monofractal analysis. For this case DFA is applied for the whole event, and no sliding windows are used. Here we can clearly observe that the majority of the events have a value of H∼0.5, indicating the presence of similar statistical processes in both type of events (QT and AT). We found average H values to be $\langle H_{QT} \rangle = 0.52 \pm 0.06$ and $\langle H_{AT} \rangle = 0.51 \pm 0.05$, implying the presence of a Brownian motion. To determine whether these QT and AT average Hurst exponents are significantly different from the null hypothesis—that the difference is due purely to randomness—we applied the students *t-test* to the distributions of the Hurst exponents. The important output of the *t-test* is the value of p, which is the probability that the difference in the means of the two distributions being compared is due to random variation. We found $p = 0.78$. This shows that the statistical differences between the sets are insignificant; although their fluctuations are very different, the overall nonlinear statistics across QT and AT are indistinguishable.

In Fig. 4, results are presented as a function of the event duration showing that intervals of different length have similar scaling exponents ranging from weak antipersistent ($H < 0.5$) to weak persistent ($H > 0.5$) fBm. The fact that most of the events fall near a random walk process ($H = 0.5$) is an indicative that long range correlations are not preserved along the time span of any particular event studied and thus become a random walk. These results were unexpected since as shown on Fig. 2, the distributions for quiet and active events encompasses marked differences as the result of different processes dominating the dynamics of the magnetosphere, i.e. during quiet times energy is stored and slowly burned keeping the magnetosphere in a relative low energy state, but during active times higher energy influx from the solar wind causes the magnetosphere to move to higher energy states where strong nonlinear processes dominate the dynamical release of energy.

4.2 Time dependent analysis

In order to find short term correlations we implemented DFA using a window size of 10000 data points that slides along the entire event series and returns a single value of H for each such window. Tests on artificial data indicate that this method allows one to find correlations that span short periods of time within the event length (Wanliss and Dobias, 2006). The results found now are quite different from the time-independent analysis; on average, we found that ATs have higher mean H values than QTs. The mean self similarity index for QTs was found to be $\langle H(t)_{QT} \rangle = 0.73 \pm 0.05$ and for ATs, $\langle H(t)_{AT} \rangle = 0.87 \pm 0.06$. This indicates that on shorter time scales long-range dependence ($H > 0.5$) is a consistent feature. Also, on average, higher correlation of the magnetic field fluctuations is expected during active magnetospheric periods. Figure 5 shows the distributions of the mean H for QTs and ATs as a function of the event length. In this case the student-t test applied to the time dependent analysis for QT and AT found $p = 2.79 \times 10^{-9}$, implying that the differences in the statistics of the averaged $H(t)$ for the computed quiet and active events are significant.

5. Conclusions

This brief communication presented an attempt to characterize the fractal behavior of the bulk magnetic field time series obtained from a single ground based observatory. Previous works reported the existence of multiscale statistics in a variety of geomagnetic indices (Vörös, 2000; Hnat *et al.*, 2003; Wanliss, 2004, 2005) and in the interplanetary magnetic field (Burlaga and Mish, 1987; Burlaga, 1991). Ohtani *et al.* (1995) and Consolini and Lui (2000) previously reported changes in statistics of the earth's magnetic field in association with different levels of magnetospheric activity. They examined the scaling properties of the magnetic field fluctuations in the magnetotail and found evidence of multifractality with a Hurst coefficient of $H \sim 0.5$ before current disruption and $H \sim 0.7$ after current disruption.

In this study we classified the data into quiet and active periods using the Kp index as the discriminator. DFA was the technique selected due to its performance dealing with nonstationary data. In terms of the monofractal ap-

proach the differences presented between QT and AT are less clear, suggesting that the fingerprints of local magnetic activity are not conserved during the time scale of a particular quiet or active event as determined by our selection criteria. On average, we found $\langle H_{QT}\rangle = 0.52 \pm 0.06$ and $\langle H_{AT}\rangle = 0.51 \pm 0.05$ for quiet and active times respectively. This is indicative that on relatively long timescales both QT and AT are uncorrelated Brownian noise and impossible to predict. This led us to consider the prospect that these data are multifractional, i.e. the scaling exponent changes as a function of time.

The possibility of correlated patches led us to investigate shorter window sizes. By sliding this window along the time series we were able to determine the temporal fluctuations in the Hurst exponent for each data set. By averaging all the H values found in each particular event we were able to distinguish clear differences in the statistical processes for both types of events (Figure 5). Results from this approach showed that both QTs and ATs have stronger correlations far from a random walk. The mean Hurst exponent $H(t)$ for quiet events was $\langle H(t)_{QT}\rangle = 0.73 \pm 0.05$ and for active events $\langle H(t)_{AT}\rangle = 0.87 \pm 0.06$. As these results show, the correlation is high while the error in the determination of $H(t)$ is about 6%, and overlap exists between the values of the temporal H for both types of event. The Students-t test returned results consistent with our expectations (i.e., quiet and active event data come from different populations). This suggests that at these high-latitudes the magnetospheric response to more intense solar wind driving is of a different type than for QT. The time-dependent, or multifractional approach, showed that the statistics of the local magnetic field are not steady and change through different levels of correlation, indicating that this correlation increases as the level of geomagnetic activity increases. It appears that the magnetic field at a single high latitude location is best described as a mfBm rather than as a fBm process. This can serve as a guide suggesting the required statistical structure for mathematical models of magnetospheric activity. We also offer a possible explanation relating the physics of QT and AT with their different Hurst exponents. Our results are consistent with Consolini and Lui (2000) who examined scaling properties of magnetic fluctuations in the magnetotail. They consistently found a lower scaling exponent before current disruption, followed by higher values afterward. They interpreted the change in scaling exponent as a reorganization during current disruption.

Further research will focus on the time where a transition from quiet to active event occurs. We do not distinguish between global and local-time effects so future studies will consider how the variability of Hurst exponent is affected by different local time selections. Whether or not local-time effects result in Hurst exponent variability does not affect our major conclusions.

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