SLR precision analysis for LAGEOS I and II

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This paper deals with the problem of properly weighting satellite observations which are non-uniform in quality. The technique, the variance component estimation method developed by Helmert, was first applied to the 1987 LAGEOS I SLR data by Sahin *et al.* (1992). This paper investigates the performance of the globally distributed SLR stations using the Helmert type variance component estimation. As well as LAGEOS I data, LAGEOS II data were analysed, in order to compare with the previously analysed 1987 LAGEOS I data. The LAGEOS I and II data used in this research were obtained from the NASA Crustal Dynamics Data Information System (CDDIS), which archives data acquired from stations operated by NASA and by other U.S. and international organizations. The data covers the years 1994, 1995 and 1996. The analysis is based on "full-rate" laser observations, which consist of hundreds to thousands of ranges per satellite pass. The software used is based on the SATAN package (SATellite ANalysis) developed at the Royal Greenwich Observatory in the UK.

1. Introduction

Satellite Laser Ranging (SLR) is a range measurement technique which uses a satellite with special mirrors (retro reflectors) and a ground station that produces short laser pulses. The range is deduced from the elapsed time of the flight for a pulse of laser light travelling from the ground station to the satellite and back again. Since its first development in the 1960s, many countries have developed and operated fixed and transportable SLR systems. Today over 35 countries cooperate in SLR activities. In this cooperation NASA plays a prominent role in coordinating international programs and improvement of laser ranging technologies; since 1990 the International Laser Ranging Service (ILRS) has officially taken this coordination role.

Today the SLR community uses many satellites to reach its objectives. The most commonly used satellites are LAGEOS I and II, ETALON I and II, ERS-1 and 2, STARLETTE, AJISAI, TOPEX/POSEIDON, STELLA, GPS-35 and 36, and several of the GLONASS satellites.

Variance component estimation was developed by Helmert in 1907 (Grafarend, 1984), and a variety of approaches have been used (e.g. Lerch, 1991; Ou, 1991). The purpose of variance component estimation is to find realistic and reliable variance components of the observations to construct correctly the a priori covariance matrix of the observations. The method divides the observations into different groups, and then simultaneously estimates the variance components for each group of observations.

Before a least square solution can be computed, the "a priori" covariance matrix has to be estimated. In SLR data processing, the a priori covariance matrix is, in general, formed in such a way that each normal point has a standard deviation computed from the statistical data compression algorithm (i.e. forming normal points using observed ranges). The standard deviations (also called normal point RMS) computed in this way reflect "internal precision", that shows the degree of consistency between only the measurements contributing to each normal point, without requiring any additional information (such as force model). On the other hand, the variance component estimation method, in one respect, reflects "external precision", which measures the comprehensive effect of all the remaining error sources.

This research uses the Helmert variance estimation method to estimate the a priori standard deviations of the global SLR stations. It has been found that the estimated standard deviations for LAGEOS I and II can range from 0.7 cm to 16 cm, while the normal point RMS range from a few millimeters to 2 cm. On the other hand, the Center for Space Research (CSR) publishes the so called "single shot RMS", which is the average RMS provided in the normal RMS (`http://ilrs.gsfc.nasa.gov/performance.html`). When compared the results published at the related web page, the single shot RMS range from 0.8 cm to 5 cm for the period of April–June 2000, excluding the Matera SLR station. The results reported in this paper, which reflect external precision, do not agree with the results reported by the CSR due to several factors, including the number of normal points at each SLR station, distribution of normal points throughout individual month, orbit integration period (one month in this research) and the orbit model (IERS92), which is the most important factor. In future analysis, the IERS96 Standards should be used.

2. Data

The SLR data used for all the studies reported in this paper are two-minute normal points of the full-rate SLR data. All the SLR data processing and variance analysis for LAGEOS I and LAGEOS II has been performed on a monthly basis, starting from January 1, 1994 to December 31, 1996. In other words a total of 72 sets of data has been analysed (36 sets for LAGEOS I and 36 sets for LAGEOS II). Observations were taken at a total of 31 fixed or mobile stations.

3. SLR Software Used at Istanbul Technical University

The SLR data analysis software used at Istanbul Technical University (ITU) is based on the SATAN (SATellite Analysis) package developed by Sinclair and Appleby at the Royal Greenwich Observatory (RGO), UK in 1986 (Sinclair and Appleby, 1986). The software runs under the UNIX operating system on HPA 2000 (Model 712/80).

There are two main programs which integrate the satellite orbit and calculate the parameters to be solved for. The first program, ORBIT, computes the satellite orbit, which is carried out using the Gauss-Jackson 8th order numerical integration method, and the partial derivatives of the satellite observations with respect to the parameters that affect the orbit such as the initial state-vector (i.e. position and velocity of the satellite), GM, satellite drag, drag rate and solar radiation pressure (Sinclair and Appleby, 1986; Sahin *et al.*, 1992). The orbital model described in IERS Standards (1993), the JGM-3 gravity field and ocean tide models are used.

Once the satellite orbit has been calculated, parameter estimation is performed by the program RGODYN. It takes each observation in turn, computes the difference between the observed and calculated range, and forms the partial derivatives of the range with respect to the parameters that affect the orbit (such as starting state-vector), parameters related to the observing station (such as coordinates) and earth rotation parameters. The partial derivatives are needed in the formation of the design matrix, which relates the parameters to be solved to the observations. RGODYN then carries out the least-squares estimation (hence computing estimates of the station coordinates and a new estimate of the starting state-vector) and also computes the unit variance factor and covariance matrix of the parameters to estimate precision.

By iterating ORBIT (using the new starting state-vector) and RGODYN until convergence is reached, a monthly solution is performed (Fig. 1). The convergence limit corresponds to corrections to the starting state-vector of the satellite and station coordinates of less than 1 cm (Sellers and Cross, 1990).

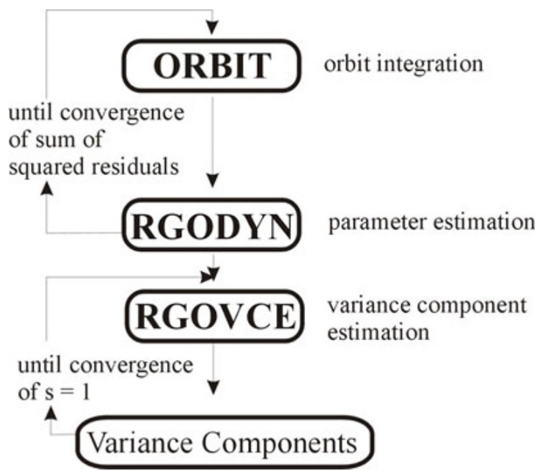

Fig. 1. ORBIT + RGODYN and RGOVCE iteration scheme.

3.1 Variance analysis

3.1.1 Variance component estimation A full derivation of the technique and computational model of variance component estimation is given in Welsch (1978), Grafarend (1984) and Sahin *et al.* (1992). Below is a summary of the mathematical models extracted from Sahin *et al.* (1992).

The Helmert equation is given by

$$\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1p} \\ h_{21} & h_{22} & \cdots & h_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ h_{p1} & h_{p2} & \cdots & h_{pp} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_p \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{bmatrix}$$

where

$$\begin{aligned} c_i &= v_i^T W_i v_i \\ h_{ii} &= n_i - 2Tr(N^{-1}N_i) + Tr(N^{-1}N_i N^{-1}N_i) \\ h_{ij} &= Tr(N^{-1}N_j N^{-1}N_i) \qquad (\text{for} \quad i \neq j) \\ s_i &= S_i^2 \end{aligned}$$

p is number of groups, n is number of observations in each group, N is global normal matrix including all observations, W_i is assigned weight matrix for ith group, N_i is normal matrix for ith group, v_i is residuals of observations, and S_i^2 is estimate of the true value of the variance factor.

The solution of the Helmert equation is iterative. Below is the summary of the estimation procedure:

1. Initial weights can be chosen to be unity for each group (i.e.: $W_1 = W_2 = \cdots = W_p = 1$).
2. Normal matrices for each group $N_1, N_2, \ldots, N_p$ and the global matrix N are formed (notice that $N = N_1 + N_2 + \cdots + N_p$).
3. Least square solutions are performed and the residuals v_i are estimated.
4. The Helmert equation is formed and the s_i are computed. Then the new weights are estimated as $W_{i+1} = W_i/s_i$.
5. If s_i is not equal to approximately 1 for all $i = 1, 2, \ldots, p$ the procedure returns to step 2.

The variance components (s_i) have only physical meaning if they are positive numbers (Sahin *et al.*, 1992). In this research, the Helmert equations always produced positive numbers.

3.1.2 Integration of VCE into RGODYN In the first step, the original SATAN package was modified for the HP 2000 UNIX WorkStation at ITU (Kizilsu, 1998). Then, after a few test solutions on HP 2000, of which results were compared and agreed with the RGO solutions, the Helmert type variance component estimation (VCE) has been implemented into the RGODYN program. The new version of RGODYN is called RGOVCE.

Besides by the input files used in RGODYN, the program RGOVCE is also controlled by a file which defines the initial standard deviations of the laser stations (equal to 1.0 for all

Table 1. Summary of the variance estimation analysis of 36 months for LAGEOS II.

ST ID	Mean standard deviation (cm)	Min. standard deviation (cm)	Max. standard deviation (cm)	Number of months contributed
1873	13.0	9.6	16.0	3
1893	4.7	4.7	4.7	1
1953	11.7	7.2	13.7	10
7080	8.6	3.0	12.6	36
7090	10.8	7.1	15.6	36
7105	9.2	3.4	14.3	30
7109	9.7	3.7	15.7	33
7110	9.6	4.0	13.3	34
7210	10.1	5.0	15.4	36
7236	9.2	6.8	14.5	7
7295	8.4	0.7	11.3	5
7308	8.1	2.9	10.2	6
7403	9.2	4.3	13.2	30
7520	5.9	3.6	8.2	2
7597	8.8	6.0	11.5	2
7805	12.4	10.9	14.7	5
7810	7.6	2.6	13.8	11
7811	6.9	3.7	12.1	15
7831	8.9	3.4	11.0	4
7835	8.5	1.6	16.9	30
7836	7.8	2.3	13.6	31
7837	9.1	3.2	13.9	17
7838	8.1	1.9	14.1	28
7839	9.0	3.4	13.3	35
7840	9.5	4.0	13.5	36
7843	9.7	3.1	15.0	33
7882	8.6	5.3	12.2	3
7883	9.3	9.0	9.6	2
7918	5.6	0.8	11.5	13
7939	11.3	6.4	14.0	29
8834	8.7	4.2	12.4	33

the stations in this research), the required number of groups (each SLR station forms one group in this research) and the name of the station in any individual group. Hence only the initial standard deviations are supplied to the program. A detailed description of the modifications to the RGODYN is given in Sahin *et al.* (1992) and Kizilsu (1998). The sequence for the variance estimation is given in Fig. 1.

One may consider that the orbit integration with ORBIT and RGODYN should be repeated in each RGOVCE run as the weights are changed, which may affect the final VCE results. This iteration has been shown not to be necessary by Sahin *et al.* (1992) since all information is stored in the design matrix and parameter changes are relatively small. Therefore the procedure in Fig. 1 has been applied in this research.

Table 2. Standard deviations (cm) of normal points for LAGEOS I and II.

	From statistical data compression (cm)						From Helmert analysis (cm)					
	LAGEOS I			LAGEOS II			LAGEOS I			LAGEOS II		
ST ID	1994	1995	1996	1994	1995	1996	1994	1995	1996	1994	1995	1996
1873	—	2.8	3.0	—	2.9	—	—	8.1	7.0	—	13.0	—
1893	—	1.5	1.2	—	1.0	—	—	7.8	5.1	—	4.7	—
1953	5.2	4.2	—	4.4	4.2	—	13.3	12.2	—	11.3	12.1	—
7080	1.4	1.3	1.2	1.3	1.3	1.2	8.2	7.9	7.9	9.0	8.0	8.8
7090	1.0	1.1	0.8	0.8	0.9	0.8	9.9	10.9	8.8	10.6	10.5	11.4
7105	1.0	0.9	1.0	1.1	0.9	1.0	9.0	8.1	6.8	9.7	8.2	9.7
7109	0.8	0.8	0.8	0.9	0.9	0.9	9.2	8.5	6.7	9.5	9.6	10.0
7110	1.3	1.0	1.1	1.0	1.0	1.1	8.1	9.5	8.0	10.0	9.8	9.3
7210	1.1	1.1	1.2	1.0	1.1	1.2	9.5	10.4	8.3	10.6	10.0	9.8
7236	1.8	—	—	2.3	1.7	1.4	7.0	—	—	10.0	6.8	10.8
7295	1.0	0.9	—	1.0	0.9	—	9.1	6.0	—	11.3	5.5	—
7308	—	1.9	4.2	—	1.8	3.6	—	—	7.0	—	9.7	6.5
7403	2.1	1.4	1.7	2.2	2.2	1.0	8.9	9.8	6.9	9.9	8.7	9.1
7520	—	3.3	—	—	3.3	—	—	6.4	—	—	5.9	—
7597	—	3.8	2.1	—	3.7	—	—	9.5	3.7	—	8.8	—
7805	7.4	6.1	8.5	7.5	6.6	8.2	12.3	12.5	14.8	12.5	13.3	11.5
7810	1.7	1.6	—	1.6	1.4	—	8.2	9.5	—	9.3	5.8	—
7811	5.4	5.4	5.6	5.4	5.1	5.6	8.6	7.9	6.4	7.0	7.1	6.6
7831	2.1	2.2	—	0.0	3.0	—	4.1	4.3	—	11.0	6.7	—
7835	0.3	0.3	0.5	0.3	0.3	0.4	9.1	8.0	6.9	8.8	8.3	8.5
7836	0.4	0.4	0.3	0.4	0.4	0.4	7.6	6.7	5.9	7.8	7.4	8.1
7837	3.1	3.9	3.2	6.7	3.4	3.7	8.4	8.3	8.5	10.3	8.7	8.4
7838	0.0	0.1	5.5	0.0	0.1	5.6	8.0	7.1	7.6	8.7	6.9	8.6
7839	1.0	0.1	0.1	0.1	0.1	0.1	7.4	10.1	8.3	8.3	9.0	9.7
7840	0.3	0.3	0.3	0.3	0.3	0.3	9.7	10.6	7.8	10.4	8.4	9.6
7843	1.4	1.1	0.8	1.4	1.0	0.9	9.8	11.2	8.1	9.1	10.4	9.5
7882	0.9	—	—	0.9	—	—	6.6	—	—	8.6	—	—
7883	1.0	—	—	1.6	—	—	7.3	—	—	9.3	—	—
7918	—	1.0	0.9	1.1	1.0	1.0	—	8.8	8.0	0.8	8.3	7.8
7939	11.6	14.0	13.6	12.6	14.5	13.4	12.3	11.5	11.5	11.5	10.8	11.7
8834	1.0	0.8	0.6	0.9	0.5	0.4	9.0	9.7	7.2	9.0	8.5	8.6

4. Estimation of Variance Components at Each SLR Station for LAGEOS I and II

As explained in 3.1, Helmert type variance component estimation divides the observations into groups. In this research, it was considered that the observations collected at each individual SLR station form their own individual group. In other words, the number of groups is equal to the number of stations in any particular month. The 1994, 1995 and 1996 data collected from LAGEOS I and II were processed on a month by month basis and the actual variance components (standard deviations in our case) were estimated using the algorithm in Fig. 1. The summary of the results for LAGEOS II only is given in Table 1. The same results for LAGEOS I are not given here in full detail, but the averages are given in following tables. Overall 31 stations contributed in this research. Although monthly standard deviations are calcu-

Table 3. A comparison between the 1987 LAGEOS I variance components (Sahin *et al.*, 1992) and the 1994, 1995, 1996 LAGEOS I variance components (standard deviations in cm).

ST ID	1987	1994	1995	1996
1873	—	—	8.1	7.0
1893	—	—	7.8	5.1
1953	—	13.3	12.2	—
7080	—	8.2	7.9	7.9
7090	9.4	9.9	10.9	8.8
7105	7.7	9.0	8.1	6.8
7109	8.1	9.2	8.5	6.7
7110	8.1	8.1	9.5	8.0
7210	8.9	9.5	10.4	8.3
7236	—	7.0	—	—
7295	—	9.1	6.0	—
7308	—	—	—	7.0
7403	—	8.9	7.0	6.9
7520	—	—	6.4	—
7597	—	—	9.5	3.7
7805	15.4	12.3	12.5	14.8
7810	7.5	8.2	9.5	—
7811	—	8.6	7.9	6.4
7831	7.9	4.1	4.3	—
7835	7.5	9.1	8.0	6.9
7836	—	7.6	6.7	5.9
7837	8.5	8.4	8.3	8.5
7838	8.8	8.0	7.1	7.6
7839	6.6	7.4	10.1	8.3
7840	8.2	9.7	10.6	7.8
7843	—	9.8	11.2	8.1
7882	—	6.6	—	—
7883	—	7.3	—	—
7918	—	—	8.8	8.0
7939	9.5	12.3	11.5	11.5
8834	—	9.0	9.7	7.2

lated, Table 1 shows only the mean standard deviations for 36 months, minimum and maximum values of standard deviations in 36 months and the number of months contributed to the solution for individual station. As seen from the third and fourth columns of the table, the standard deviations range from 0.7 cm (7295) to 16 cm (1873).

The variance component estimation method assumes that all of the unmodelled errors are purely stochastic with a continuous probability distribution function and zero mean (Sahin *et al.*, 1992). This is not the case with SLR processing, since range residuals are a function of both the quality of observations and the orbit integration. If it is supposed that there is a range bias for a station, the range residuals should reflect this. For instance, there should be a jump between the residual distributions of the two passes. The Helmert method can be applied to the global solution together with the estimation of the range biases. The mean of the residuals tends to zero and the associated standard deviation gets smaller when increasing the number of parameters to be solved for, including the range biases and variance components (Sahin *et al.*, 1992).

Here we are trying to estimate the quality of the observations. The quality of the orbit depends on the force models and the data distribution over the time of the integration. In this case, the SLR stations do not collect data simultaneously. Some of the stations may only have a few days of observations while some have observations throughout the whole month.

5. Comparisons

5.1 Comparisons of standard deviations derived from the Helmert analysis and the statistical data compression

Individual ranges to LAGEOS I and II can be measured with a precision of less than 1 cm. (Anon, 1999). However, in SLR data processing, normal points are used, which are derived from the statistical compression of the measured ranges. Table 2 shows two values: one is the standard deviations of the normal points from the statistical data compressing process, and the other from the Helmert analysis. As seen from the table, the standard deviations computed from the statistical data compressing process are below the 10 mm level except for a few stations (i.e. 1953, 7403, 7805, 7811, 7831, 7837, 7939). However, the Helmert standard deviations, ranging between 4 and 15 cm, do not agree at all with the statistical data compressing results. As the Helmert standard deviations are computed from the global SLR data, they show the precision of the orbit determination as well. There should be more comments on these differences to be discussed. In addition to that, the models used in orbit determination and range measurements do not perform well.

5.2 Comparisons of the 1994, 1995, 1996 variance components with 1987 for LAGEOS I

The 1987 SLR variance components for LAGEOS I were processed by Sahin *et al.* (1992). Table 3 shows the mean of the variance components of twelve months in terms of standard deviations for the years of 1987 and 1994, 1995, 1996. As seen from the table, the standard deviations, on average, have not improved significantly. Sometimes they remain the same and sometimes they even get worse. For instance the standard deviations at 7805, 7831 and 7838 have improved, but not significantly. These results show that there are still problems with SLR data processing (maybe in the SATAN package), which need to be worked out.

6. Conclusions

The Helmert type variance component estimation method has been applied to the SLR data collected from January 1, 1994 to December 31, 1996. In other words 36 months of data for LAGEOS I and 36 months of data for LAGEOS II have been analysed on a monthly basis, and the actual standard deviations (not normalised) have been estimated. The overall research has shown that the LAGEOS I data quality is almost the same as LAGEOS II. The results also indicate that the standard deviations derived from the Helmert method are at decimeter level. However, the RMS of normal points computed by the SLR stations is at a few millimeters level. It should be noticed that the Helmert methods derive the standard deviations (or variance components) from the least square adjustment of the global SLR network, whereas the RMS of the normal points computed by the SLR stations, which show internal precisions, are obtained by the statistical data compression algorithms. Therefore it is not absolutely right to compare these two results.

The standard deviations from 1994, 1995, 1996 have also been compared with the previously analysed 1987 LAGEOS I data. From this comparison, it was seen that there is not much improvement in standard deviations. This is unlikely to be the case in practice, since most of the SLR systems have been improved by the owners. However, there should be other factors which cause not good standard deviations. For instance, distribution of the observations during the orbit integration, the orbit and measurement models used in the SLR processing can affect the standard deviations estimated by the Helmert method.

It is, mathematically, quite difficult to implement the Helmert method into any SLR data processing software. The SATAN package used in this research is almost the simplest software available within the SLR community. On the other hand, there is no doubt of the performance of the Helmert method as justified by several authors in different areas. In addition, the SLR is almost the most accurate technique in point positioning within the area of satellite geodesy. So the results derived with the SATAN package, which is a sort of status report, encourage us to improve the computational model for the orbit in order to have better post-fit residual RMS values (note that post-fit RMS results derived from the SATAN package are not given in this paper). The Helmert technique should then be applied to more recent SLR data, taking into account the range biases.

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