Long term probability of a Magnitude 8 Kanto earthquake along the Sagami Trough, central Japan

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We attempt to estimate the long-term probability of a Magnitude (M) 8 earthquake along the Sagami Trough in the Kanto subduction zone, central Japan. A Brownian passage time model is applied to sets of historical earthquakes identified in previous studies. An optimal model is obtained by the maximum likelihood method for each data set. The optimal parameters are not well constrained since each data set includes a small number of earthquakes. To obtain reliable probabilities, two weighting methods are introduced. First, we apply the weighted log-likelihood method, where the model parameters are estimated from the log-likelihood function, summed up with each log-likelihood weighted in proportion to the reliability of the data set. Second, probabilities are estimated as the weighted average of every alternative model. The weight of each model represents the normalized relative likelihood of the model. The probabilities of the weighted log-likelihood function are within the ranges of those obtained for each set. In averaging the proposed sequence and over probable parameter values, the probability of an M 8 earthquake occurring in the next 30 years is estimated to be 2.0 to 4.6%, depending on the cutoff value of the weight.

Key words: Brownian passage time model, weighted log-likelihood method, Kanto, M 8 earthquake, Japan.

1. Introduction

The 1923 and 1703 earthquakes in the Kanto region, central Japan, are Magnitude (M) 8 class earthquakes that were caused by the relative motion between the Philippine Sea Plate (PH) and the North American Plate (NA) along the Sagami Trough, where the PH is subducting beneath NA.

According to the Earthquake Research Committee (ERC), a committee of the government of Japan, the long-term probability of an M 8 earthquake during the next 30 years in Kanto, central Japan, is no higher than two percent. This probability is estimated using a Brownian passage time (BPT) model with two model parameters, which are assigned based on geological and geomorphological evidence, and model parameters for other areas such as off Miyagi Prefecture, the Nankai and the Tonankai areas, and major inland Quaternary active faults. Efforts have been made to produce more reliable models by using different kinds of data simultaneously (Fitzenz *et al.*, 2010; Rhoades and Van Dissen, 2003). In this paper, we discuss the validity of the current BPT model for Kanto by using historical earthquake sequences proposed by Ishibashi (1994), Shishikura (2003), and Shimazaki *et al.* (2011a, b). In each of these sequences, only a small number of earthquakes, specifically five (Ishibashi, 1994), four (Shishikura, 2003), and three (Shimazaki *et al.*, 2011a, b), are listed. Therefore, we could not constrain the BPT model parameters well, but probabilities for the next 30 years could be estimated by taking the average of the probable hazard function and comparing this with the currently reported value.

2. BPT Model and Parameter Uncertainties

The probability density function of the time intervals between successive events in the BPT model (Matthews *et al.*, 2002) is given as:

$$f(t|\mu,\alpha) = \sqrt{\frac{\mu}{2\pi\alpha^2 t^3}} \exp\left(-\frac{(t-\mu)^2}{2\mu\alpha^2 t}\right)$$

where μ and α are model parameters. The parameter μ indicates the average recurrence interval and the parameter α concerns the variance of recurrence intervals. Observing the time intervals (t_1, t_2, .. t_n), the log-likelihood l is given by

$$l(t_1, t_2, ..t_n|\mu,\alpha) = \sum_{i=1}^{n} \log_e f(t_i|\mu,\alpha)$$

The optimal parameters (μ_m and α_m) can be estimated by the maximum likelihood method.

In an exact sense, we should take into the above log-likelihood function a factor corresponding to non-occurrence after the 1923 earthquake, which is represented by the reliability function (survivor function, complementary cumulative distribution function) at the time point of interest (Ogata, 1999). Only about 90 years have passed since the last event and the reliability, even in the smallest case, becomes 0.996, which reduces only 0.004 units of log-likelihood. For simplicity, this factor is neglected in our study.

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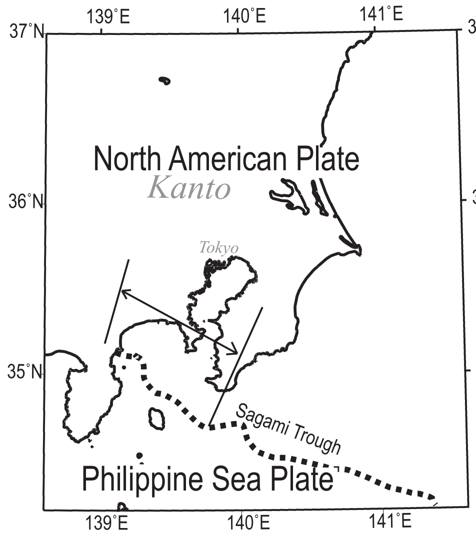

Fig. 1. Map of the Kanto region, central Japan. The source area of an *M* 8 Kanto earthquake presumed in the present study is indicated by an arrow.

Table 1. Historical earthquake sequences used in the study. The optimal BPT parameters are listed in the second set. The AIC's values for the optimal case and the Poisson model are in the third set. The difference in AIC, specifically the AIC of the Poisson minus that of the optimal BPT, is also given. Values of log-likelihood for the Poisson and the optimal BPT are listed in the bottom set. At the bottom, Δl refers to a value of the log-likelihood of the Poisson minus that of the optimal BPT.

	Ishibashi (1994)	Shishikura (2003)	Shimazaki *et al.* (2011b)	Weighted log-likelihood
878/Nov./1	○	○		○
1293/May/27	○	○	○	○
1433/Nov./6	○			○
1703/Dec./31	○	○	○	○
1923/Sept./1	○	○	○	○
BPT optimal values				
μ	261.21	348.28	315.14	302.70
α	0.40	0.31	0.32	0.38
AIC				
Poisson	54.52	43.12	29.01	42.28
BPT optimal case	51.68	40.09	27.81	40.46
ΔAIC	2.84	3.03	1.20	1.82
ΔAIC/2	1.42	1.52	0.60	0.91
Log-likelihood				
Poisson	−26.26	−20.56	−13.51	−20.14
BPT optimal case	−23.84	−18.05	−11.91	−18.23
Δl	−2.42	−2.52	−1.60	−1.91

In selecting the better model, the Akaike Information Criterion (AIC, Akaike, 1977; Sakamoto *et al.*, 1983) is widely used, which is defined in our case as

$$\mathrm{AIC}(\mu, \alpha) = -2 \cdot l(\mu, \alpha) + 2 \cdot K,$$

where K is the number of free model parameters and equals one for the stationary Poisson model and two for the BPT model. A smaller AIC implies a better model.

When the optimal parameters are not well constrained, a robust estimate of a practical value can be obtained by averaging such values over models with different weights in proportion to their likelihood (Akaike, 1973; Rhoades *et al.*, 1994; Ogata, 2002). For a model with parameters (μ, α), the weight $w(\mu, \alpha)$ is given by:

$$w(\mu, \alpha) = \exp\{\Delta l(\mu, \alpha)\},$$
$$\Delta l(\mu, \alpha) = l(t_1, t_2, ..t_n|\mu, \alpha) - l_m(t_1, t_2, ..t_n|\mu_m, \alpha_m),$$

The average probability, P_a is given by

$$P_\mathrm{a} = \frac{\sum P(\mu, \alpha)w(\mu, \alpha)}{\sum w(\mu, \alpha)}.$$

In the present case, we consider the 30-year probability as the practical value. The summation in the above equation is executed within a cutoff value of Δl. The method presented here is similar to the Bayesian average described in Rhoades *et al.* (1994), Ogata (2002) and others, but the distinction is that we use only parameters of log-likelihood values higher than a certain threshold level.

3. Historical Earthquakes

Only two earthquakes, in 1923 and 1703, are widely accepted as *M* 8 earthquakes in Kanto, which were caused by the relative motion of the Philippine Sea Plate subducted beneath the North American Plate (Eurasia Plate, Okhotsk Plate) along the Sagami Trough (Fig. 1). Some other preceding historical earthquakes have been nominated as *M* 8 Kanto earthquakes, with different authors reporting different sequences. Table 1 lists three different historical sequences used in the present study (Ishibashi, 1994; Shishikura, 2003; Shimazaki *et al.*, 2011a, b). Shishikura (2003) suggested two possible cases; one is listed here and the other includes an event of 1257 instead of 1293. In the present study, we adopted only the former case since the differences in model parameters between them are negligible compared with those among the three in Table 1.

4. Log-likelihood and Probabilities

Figures 2(a)–(c) illustrate the contours of log-likelihood within certain ranges of the two model parameters. It is obvious that the values used in estimating the current probability publicly reported (ERC, 2012), depicted by the shaded zone, includes none of the three sets of optimal values and, worse, that it includes probable sets of parameters even less than −2.0 in log-likelihood. The differences in AIC (ΔAIC) and log-likelihood (Δl) between the optimal model and the stationary Poisson model are listed in Table 1. Considering the difference in the number of model parameters between them, the Poisson model is better fitted to the historical sequences than the BPT model in some parts of the shaded zone.

Table 2 lists the probability of an *M* 8 earthquake in the next 30 years for each proposed sequence. 100,000 sets of BPT parameters within the contour of −2.0 were randomly generated and the Akaike weight was applied to estimate the average value. The first row gives the values estimated with the optimal parameters for each sequence. The three sequences share the common feature that the probability increases with averaging over a wider range (less optimal case). The sample size of each sequence is so small that the

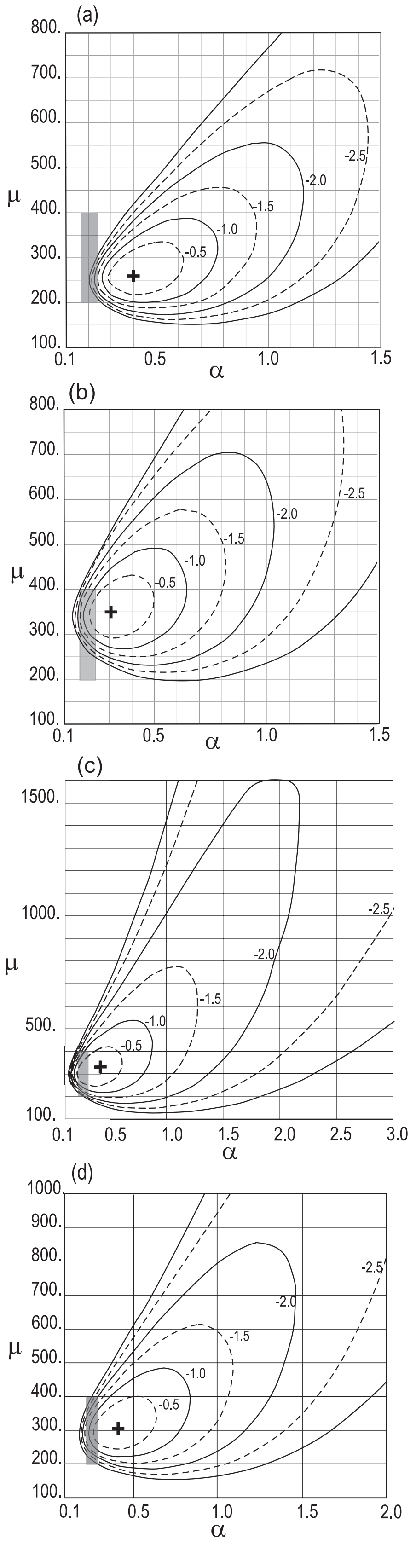

Fig. 2. (a–d) Contour maps of log-likelihood values for the BPT model parameters. The cross indicates the position of the maximum likelihood values. The shaded zone indicates the range of model parameters used in estimating the long-term probability issued by the ERC: (a) for the sequence proposed by Ishibashi (1994), (b) Shishikura (2003), (c) Shimazaki *et al.* (2011b), (d) Weighted log-likelihood.

Table 2. Probability of an *M* 8 earthquake in the next 30 years for various cases. Probabilities listed in the first row indicate those for the optimal BPT cases. Probabilities averaged with weighting down to $\Delta l = -2$ at every 0.5 step are given in the following lines.

	Ishibashi	Shishikura	Shimazaki *et al.*	Weighted log-likelihood
Optimal case	0.027	0.000	0.001	0.007
$\log_e$ (weight)				
$0 \sim -0.5$	0.041	0.002	0.011	0.020
$0 \sim -1$	0.050	0.007	0.024	0.031
$0 \sim -1.5$	0.056	0.012	0.036	0.040
$0 \sim -2$	0.060	0.019	0.045	0.046
Poisson	0.109	0.082	0.091	0.094

difference in log-likelihood between the optimal case and the other cases gradually increases with distance from the optimal point.

5. Weighted Log-likelihood

The probabilities in Table 2 vary for different authors. In averaging the probabilities among the three, we have attempted to apply the weighted log-likelihood method (Wang and Zidek, 2005). In this method, the model parameters are estimated from the log-likelihood function, summed up with each log-likelihood weighted in proportion to the reliability of the data.

The historical sequences adopted here are reported by the authors from different viewpoints with independent evidence: historical documents, or geological or paleoseismological evidence. However, times of earthquakes are precisely dated in historical documents, but the assignment of the events as a Kanto recurrent earthquake is the point of argument. For this reason, we do not apply the method handling uncertainties proposed in early studies (Rhoades *et al.*, 1994; Ogata, 1999) but the method of weighted log-likelihood.

We adopt an equal weight for the sequences, while requiring a total weight equal to one. Thus, the weighted log-likelihood function, l_w is given as:

$$l_w = \frac{1}{3}(l_1 + l_2 + l_3),$$

where the subscripts, 1, 2, and 3 refer to the log-likelihood for the sequences by Ishibashi (1994), Shishikura (2003), and Shimazaki *et al.* (2011a, b).

The estimated model parameters and probabilities are listed in the last column of Tables 1 and 2. Comparing these values with those obtained for each sequence, the values estimated from the weighted log-likelihood function are within the ranges of the corresponding estimates. Therefore, it is reasonable to consider that these values represent the results obtained when the BPT model is adapted to an *M* 8 sequence in Kanto with largely uncertain historical data.

It can be seen in Table 2 that the probabilities increase with lower cutoff values of Δl. In Fig. 3, the average probability in each 0.1 unit of Δl was examined to estimate the lower limit of the value. The solid line indicates the probability averaged over the respective Δl and the dashed line

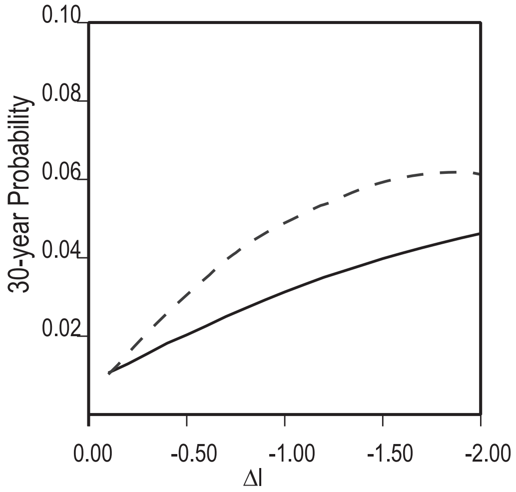

Fig. 3. Probability of an M 8 earthquake in the next 30 years. The probability averaged down to the respective weight is indicated by the solid line. The probability averaged over 0.1 units at the respective point is indicated by the dashed line.

indicates the probability averaged over 0.1 units at the respective point. It is likely that the probability averaged over 0.1 units reaches a maximum of about 0.06 at a point between 1.5 and 2.0. Therefore, the average probability over probable parameters becomes less than 0.06. A BPT model with handling model parameter uncertainties and averaging uncertain sequences suggests that the probability of an M 8 earthquake in the next 30 years in Kanto can be estimated at somewhere between 0.02 and 0.046 and cannot exceed 0.06.

At the bottom of Table 2, the probabilities estimated with the Poisson model are listed. Comparing each of these values with the respective ones of the BPT model, the estimates of the Poisson models are about two times larger than those of the BPT model.

6. Discussion and Summary

Considering that the current probability issued to the public was estimated by including parameter values of Δl ranging down to -2.0 (Figs. 2(a–d)), the probability of 0.046 for the weighted log-likelihood case would become an alternative to the current probability. The difference between the publicized value, and that of the present study, stems from differences in the model parameters, which are constrained to a value in the range 200 to 400 for μ and 0.17 to 0.24 for α in the former case. The optimal values of parameter α (Table 1) are obviously larger than that used in the ERC estimation. It partially contributes to larger probabilities than that of the ERC estimation (Ishizeki and Kumamoto, 2007).

The recent great earthquake in Tohoku, northeast Japan ($M_w = 9.0$, March 11, 2011), will definitely disturb the regularity of the recurrent sequences within its source and nearby areas. For example, the source area of the off-Ibaraki earthquake (M 7.0) on May 8, 2008, is likely to be ruptured again only about three years after the last event, where values of 22 years and 0.20 have been adopted as the BPT parameters. This great earthquake suggests that the current model is applicable only under limited conditions, and that having the α parameter range from 0.17 to 0.24 will be useful in these limited cases. Without this prior information, the average probability over probable parameters would become an alternative estimate to the current one.

In summary, we conclude that the BPT model is slightly superior to the stationary Poisson model based on historical earthquakes, and geologic and paleoseismological evidence, in Kanto. In averaging the proposed sequence and the average probability over probable parameters instead of constraining the α parameter, a probability of 2.0 to 4.6% could be obtained, depending on the cutoff value of the probable parameters.

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