Numerically modelling the ratio of cross-strait voltage to water transport for the Bering Strait

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We find out if the ratio of cross-strait voltage to water transport for the Bering Strait (BS) is constant. For this purpose, we have developed a technique to construct BS water velocity maps. We have built up a three-dimensional conductivity model for the BS region. Using this model, we have simulated coast-to-coast voltage for the various velocity maps constructed. We have found that the voltage/transport ratio remains constant for electromagnetic field periods exceeding 2 days. We have estimated the ratio as 239 ± 11 mV/(km$^2 \cdot$ m/s) assuming bottom sediment conductance to be 600 S. We conclude that measuring cross-strait voltage allows the monitoring of BS water transport.

1. Introduction

Water transport through the Bering Strait (BS) is an important climate-creating factor in the Polar zone. Sanford and Flick (1975), Robinson (1976), Larsen and Sanford (1985), Baines and Bell (1987), Larsen (1992), Shneyer *et al.* (1994) have demonstrated that to estimate water transport T, use may be made of cross-strait voltage U, which is due to a sea water stream flowing through the planetary magnetic field. Although, theoretically speaking, voltage U depends on the stream velocity distribution $\mathbf{v}(x, y)$, it is generally assumed to be directly proportional to water transport T;

$$U = \gamma\, T. \tag{1}$$

Here the coefficient γ is constant for all actual stream velocity distributions. In this case one can readily determine T by measuring U given that the coefficient γ is known. However, the coefficient γ happens to vary significantly with meandering.

In this paper, we verify that a constant coefficient γ does exist for the BS. For this purpose, we first constructed a three-dimensional (3-D) conductivity model for the BS region. Then we developed an incompressibility-based technique which, on account of some BS velocity measurements from elsewhere, yielded possible stream velocity distributions. Assuming the conductivity model and the resulting velocity distributions, we performed numerical simulations of the corresponding electromagnetic (EM) field.

Our conclusion is that for EM field periods exceeding 2 days, the voltage/water transport ratio is actually constant and accounts for 239 ± 11 mV/(km$^2 \cdot$ m/s).

2. Modelling the Stream Velocity

To construct a physically realistic depth-integrated stream velocity $\mathbf{v}(x, y)$, we apply the rigid lid stream incompressibility equation

$$\nabla \cdot \mathbf{w}(x, y, z) = 0,$$

where $\mathbf{w}(x, y, z)$ is the volume stream velocity. Integrating this equation over z from the surface ($z_s = 0$) to the Strait bottom ($z_b = h(x, y)$) and taking into account the boundary conditions $w_z(z_s) = w_z(z_b) = 0$, we have

$$\nabla_\tau \cdot \mathbf{v}(x, y) = 0. \tag{2}$$

Here we have determined the depth-integrated stream velocity $\mathbf{v}(x, y)$ as

$$v_x(x, y) = \int_0^{h(x,y)} w_x(x, y, z)dz,$$
$$v_y(x, y) = \int_0^{h(x,y)} w_y(x, y, z)dz,$$

and introduced the following notations: $h(x, y)$ is the depth of the Strait, $\nabla = (\partial_x, \partial_y, \partial_z)$ is the volume gradient, ∇_τ is the horizontal gradient; x, y, z are the rectangular coordinates directed eastwards, southwards and vertically downwards, respectively. Following Eq. (2), we express the depth-integrated velocity $\mathbf{v}(x, y)$ of any incompressible stream as

$$\mathbf{v}(x, y) = -\phi(p(x, y))\ \mathbf{e}_z \times \nabla_\tau p(x, y), \tag{3}$$

where $\mathbf{e}_z$ is the downward unit vector, and the sign "×" denotes the vector product. Here the potential function $p(x, y)$ defines the stream direction. More precisely, the equation $p(x, y) = Const$ describes the stream lines. The function $\phi = \phi(p)$ determines the magnitudes of the stream velocity.

For any given potential function $p(x, y)$ and for the velocity $\mathbf{v}(x, y)$ known at experimental points (x_i, y_i), $i = 1, \ldots, N$, we find $\phi(p(x_i, y_i))$ using Eq. (3). Knowing the function ϕ at points $p_i = p(x_i, y_i)$, we interpolate $\phi(p)$ for all p and, again using Eq. (3), obtain velocity $\mathbf{v}(x, y)$ everywhere, with continuity of the potential function $p(x, y)$ being provided by means of appropriate interpolation.

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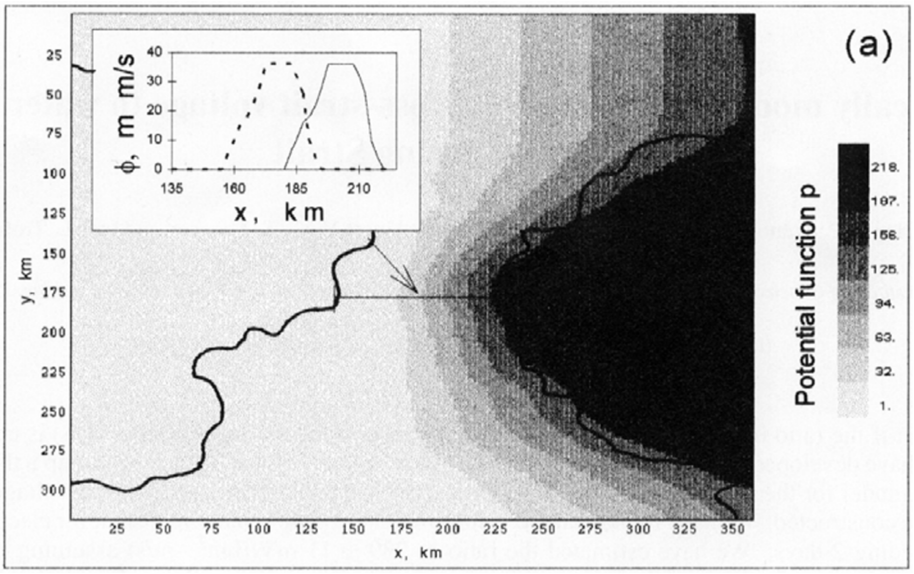

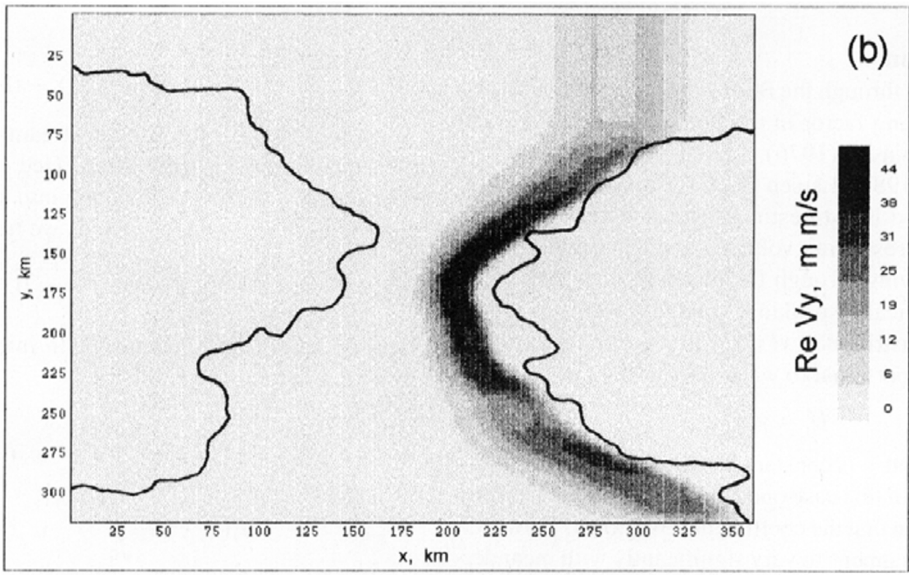

Fig. 1. (a) The "Big" background picture is the potential function $p(x, y)$ which defines the stream direction within the BS region, the "small" foreground picture is the function $\phi(p)$ along a latitudinal profile crossing the BS. The location of the profile is presented by a segment of the straight line on the background picture. The y-coordinate of the profile is 167.5 km, the x-coordinates of the profile ends are 135.0 and 225.0. The solid curve represents $\phi(p)$ for the "eastern stream" (closer to the Alaska coast), the dashed one shows $\phi(p)$ for the "western stream" (closer to the Chukotka coast); (b) $v_y(x, y)$ associated with the "eastern stream"; (c) $v_y(x, y)$ associated with the "western stream"; (d) conductance map for the BS region.

We built a potential function $p(x, y)$ for the BS on the basis of the *Atlas of the Oceans* of the Navy Oceanographic Survey of the USSR (1980) (see the background picture of Fig. 1(a)). Then we found the function $\phi(p)$ (see Fig. 1(a), the solid curve) on account of the experimental velocity measurements by Bloom (1964), the *Atlas* as well, and the bathymetry map of the BS (Navy Oceanographic Survey of the USSR, 1972). From ϕ and p, we got the velocity distribution $\mathbf{v}(x, y)$ which we designated velocity of the "eastern stream" (see Fig. 1(b)). To model stream meandering, we shifted the function $\phi(p)$ 25 km westwards (see Fig. 1(a), the dashed curve). From an unchanged potential function $p(x, y)$ and from the shifted function $\phi(p)$, we obtained some new velocity distribution $\mathbf{v}(x, y)$ which we naturally call the velocity of the "western stream" (see Fig. 1(c)).

Water transport through the BS

$$T = \int \mathbf{v} \cdot \left(\mathbf{e}_z \times d\mathbf{l} \right) \tag{4}$$

happened to be 0.837 $\text{km}^2 \cdot \text{m/s}$ for the "eastern stream" and 0.865 $\text{km}^2 \cdot \text{m/s}$ for the "western" one (in Eq. (4) integration was done from one coast to the opposite). These figures appeared to be very close to the mean values for water transport calculated by Roach *et al.* (1995) on account of the direct hydrologic measurements.

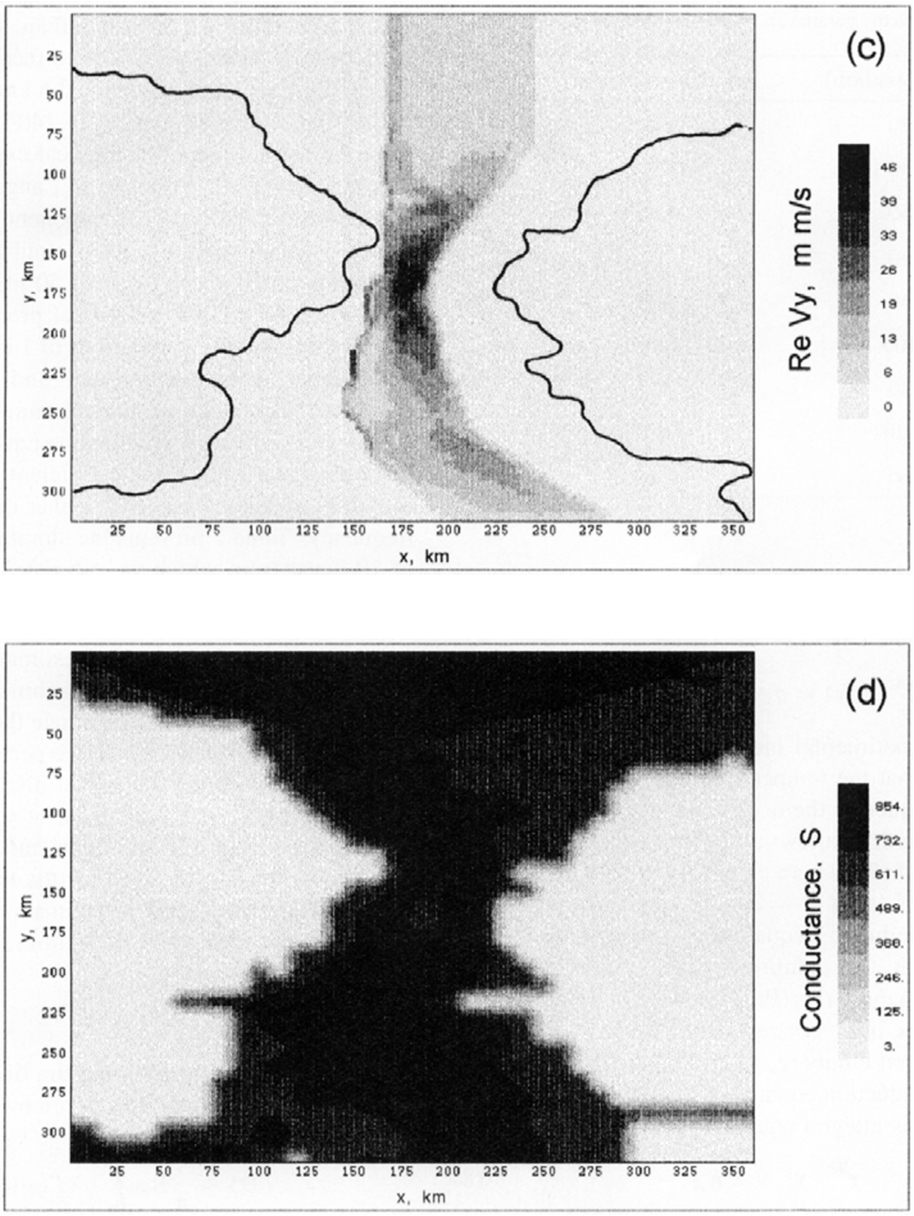

Fig. 1. (continued).

3. Cross-Strait Voltage Modelling Technique

To model cross-strait voltage between probe grounding sites

$$U = \int \mathbf{E} \cdot d\mathbf{l}, \tag{5}$$

let us consider the BS conductivity model, which consists of a thin surface layer, as being in galvanic contact with underlying one-dimensional (1-D) earth. This thin inhomogeneous layer has conductance $\tau(x, y)$ and approximates strait water, the bottom sediments and inland sediments. Conductance of the Strait is defined as water conductivity $\sigma_s = 3.4$ S/m multiplied by the BS depth $h(x, y)$ taken from a bathymetry map. Following Korotaev *et al.* (1981), we take the bottom sediments conductance to be 600 S. The inland conductance is assumed to be 3.4 S everywhere because no inland conductance maps are available. Figure 1(d) presents the resulting conductance map for the BS region. Conductivity $\sigma_n(z)$ of underlying 1-D earth is presented in Table 1. With z ranging from zero down to 300 km, conductivity $\sigma_n(z)$ values are borrowed from Moroz (1991) who has constructed geoelectric section for the Kamchatka region. Deeper $\sigma_n(z)$ distribution follows the results of global geomagnetic sounding by Zinger *et al.* (1993).

Electric field $\mathbf{E}(x, y, z)$, necessary to calculate (5) voltage, satisfies the induction equation

$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu_0\sigma_n(z)\mathbf{E} = -i\omega\mu_0\Big(\tau(x, y)\mathbf{E}_\tau + \mathbf{j}^{\text{ext}}\Big)\delta(z), \tag{6}$$

where $\omega = 2\pi f$ is the angular time frequency; μ_0 is the magnetic permeability of the vacuum; and $\delta(z)$ is the Dirac's

Table 1. The geoelectric parameters of 1-D earth within the BS region.

Thickness (km)	Resistivity (Ohm m)
20	600
30	100
60	800
50	30
40	50
100	100
80	32
320	8
∞	0.2

delta function. Here the electric current $\mathbf{j}^{\text{ext}}$ is determined according to Ohm's law for a stream moving in the presence of planetary magnetic field $\mathbf{B}^0$ as

$$\mathbf{j}^{\text{ext}}(x, y) = \sigma_s \mathbf{v}(x, y) \times \mathbf{B}^0. \quad (7)$$

Note that the experimental measurements by Roach *et al.* (1995) showed that the temperature and salinity in the BS (and thus, consequently the ocean conductivity) vary unappreciably with depth. It allows us to represent electric current $\mathbf{j}^{\text{ext}}$ in the form of (7), where conductivity σ_s is assumed to be independent of depth.

To solve the induction equation (6), we use an integral equation approach. Our solution is based on a modified formulation of Neumann series (Pankratov *et al.*, 1995), but otherwise follows the basic concept of the iterative-dissipative method (Singer and Fainberg, 1985; Singer, 1995). The approach reduces induction equation (6) to an integral one of a specific kind. This integral equation reads

$$\boldsymbol{\chi} = \boldsymbol{\chi}^0 + KR\boldsymbol{\chi}, \quad (8)$$

where

$$\boldsymbol{\chi} = \frac{1}{2\sqrt{\tau_0}}\Big((\tau + \tau_0)\mathbf{E} + \mathbf{j}^{\text{ext}}\Big), \quad (9)$$

$$\boldsymbol{\chi}^0 = K\frac{\sqrt{\tau_0}}{\tau + \tau_0}\,\mathbf{j}^{\text{ext}}, \qquad R = \frac{\tau - \tau_0}{\tau + \tau_0}, \qquad K = 2\tau_0 G^e + I.$$

Here I denotes the identity operator, G^e specifies the electric Green operator of a 1-D reference model which includes layered earth of conductivity $\sigma_n(z)$ and a thin homogeneous surface layer of conductance τ_0; the methodology of constructing the G^e operator and its explicit form are presented in Avdeev *et al.* (1997). For integral equation (8), the solution $\boldsymbol{\chi}(x, y, z)$ can be written in a form of an always convergent Neumann series expansion

$$\boldsymbol{\chi} = \boldsymbol{\chi}^0 + (KR)\boldsymbol{\chi}^0 + (KR)^2\boldsymbol{\chi}^0 + \cdots.$$

Then, knowing $\boldsymbol{\chi}$, we determine the electric field $\mathbf{E}(x, y)$ from Eq. (9). Now we compute voltage U from Eq. (5). Details of the approach and examples of the code usage are to be found in Pankratov (1991), Fainberg *et al.* (1993), Avdeev *et al.* (1997).

4. Estimating the Meandering Effect

To estimate the meandering effect, we simulated the EM field in the BS model with a thin surface layer discretized into 144 × 128 cells, each cell being 2.5 km × 2.5 km in size. To find $\mathbf{j}^{\text{ext}}$ we synthesized $\mathbf{B}^0 = (-14042, 3442, -52747)$ nT from the coefficients of a spherical expansion of the planetary magnetic field presented in Langel (1992). Our modellings left out the vertical component of $\mathbf{j}^{\text{ext}}$ since its effect is negligible in such a shallow strait as the BS where the maximum depth doesn't exceed 50 m. We obtained cross-strait voltages for five latitudinal profiles crossing the BS performing modelling at periods of 1 hour, 12 hours, 2 days and 10 days for both the "western" and the "eastern streams". Each modelling run took about 4 minutes on a Pentium 100 MHz. Our numerical simulations have shown that at periods outpacing 2 days, the coefficient γ, with an accuracy of 4–6%, appears to depend neither on meandering nor on frequency. Table 2 presents the simulation results at period of 10 days, from which we estimated the coefficient γ as 239 ± 11 mV/(km$^2 \cdot$ m/s). This estimate refers to the profile with minimal distortions caused by meandering.

It is clear that the numerical estimate of γ must strongly depend on the value of bottom sediments incorporated into the conductivity model. To evaluate the dependence quantitatively, over a 10 day period, we performed additional numerical modelling for the model discounting bottom sediments. In this case, the estimate of γ for the same profile happened to be 746 ± 78 mV/(km$^2 \cdot$ m/s) which is three times as much as the one obtained for the model considering the bottom sediments. Thus, to estimate reliably coefficient γ for the BS, one must know the bottom sediment conductance as accurately as possible.

5. Discussion

5.1 Grounds for monitoring the BS water transport

In view of slow water transport temporal changes, we can write

$$T(t) \approx \frac{1}{\pi}\text{Re}\int_0^{1/t_1} T(\omega)\mathrm{e}^{-i\omega t}d\omega,$$

where t_1 is the minimal period of the transport changes. It follows from Eqs. (6) and (7) that the coefficient $\gamma(\omega, \mathbf{v})$ can be introduced as

$$U(\omega) = \gamma(\omega, \mathbf{v})\ T(\omega). \quad (10)$$

If the coefficient γ doesn't depend on frequencies $\omega \ll 1/t_1$ and on various realistic distributions of velocity $\mathbf{v}$,

$$\gamma(\omega, \mathbf{v}) \approx \gamma = Const, \quad (11)$$

the inverse Fourier transform of Eq. (10) will yield

$$U(t) \approx \gamma\ T(t).$$

By numerical simulations for the two distributions $\mathbf{v}(x, y)$ associated with the stream meandering, we show that γ is held almost constant for periods exceeding 2 days. This means that the given bottom sediment conductance for the BS, voltage U allows us to monitor water transport T averaged over a length of time overpacing 2 days.

Table 2. The ratio γ of BS cross-strait voltage U to water transport T modelled for the "eastern" and "western" streams.

y-coordinate of the profiles (km)	x-coordinates of the profiles ends (km)	γ (mV/(km^2 · m/s)) for the "eastern stream"	γ (mV/(km^2 · m/s)) for the "western stream"
157.5	145.0–227.5	251.2	229.8
162.5	137.5–227.5	252.1	228.2
167.5	135.0–225.0	250.6	228.6
172.5	137.5–230.0	247.7	226.7
177.5	137.5–227.5	250.1	227.3

5.2 Suppressing the voltage distortions

In a general case, the cross-strait voltage caused by the stream is distorted by tidal-induced, geomagnetic-induced voltages (Larsen, 1992) and by global oceans-induced voltages (Tyler *et al.*, 1997). Since the tidal-induced voltage has periods not exceeding 1 day, we can suppress it by low-frequency filtering with a cutoff period of 1.5 days. Moreover, the filtering should cancel solar diurnal variations as well. However, in the BS region, the auroral jet-induced voltage might be relevant for periods over 2 days.

To suppress the auroral jet-induced voltage, we propose to use the following conventional scheme:

1. We measure the magnetic field $\mathbf{H}(t)$ at a reference site inland synchronously with the cross-strait voltage $U(t)$, and calculate its Fourier transform $\mathbf{H}(\omega)$.

2. We build a digital filter $\mathbf{L}(\omega)$, by numerical simulations for a model with two thin inhomogeneous layers (Avdeev *et al.*, 1995), where the upper thin layer incorporates the auroral jet. More precisely, we compute the magnetic field $\mathbf{H}_m(\omega)$ at the reference site and the cross-strait voltage $U_m(\omega)$ caused by the auroral jet having a realistic geometry but unit amplitude. Then we define the filter as $\mathbf{L}(\omega) = U_m \dfrac{\mathbf{H}_m}{|\mathbf{H}_m|^2}$.

3. Further, we obtain the auroral jet-induced voltage $U_j(t)$ as the inverse Fourier transform of $U_j(\omega) = \mathbf{L}(\omega) \cdot \mathbf{H}(\omega)$.

4. Finally, we estimate the stream-induced voltage to be sought as $U_{\text{stream}}(t) = U(t) - U_j(t)$.

To suppress the global oceans-induced voltage, the same procedure is valid.

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