Earth Planets Space, **50**, 723–732, 1998

On the characteristics of successive geomagnetic jerks

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(Received February 2, 1998; Revised August 24, 1998; Accepted August 25, 1998)

Spherical harmonic models of the 1969, 1979 and 1992 geomagnetic jerks are computed using data from about 160 worldwide geomagnetic observatories. The dominance of the internal origin part with respect to the external one confirms again the internal origin of these events. A change of sign is observed between two successive jerks (1969–1979, 1979–1992). The acceleration jump of the fluid flow at the core mantle boundary (CMB) generating the three jerks is computed. Striking similarities between the three acceleration maps are observed (within the sign change mentioned above). These results suggest some long time scale memory in the processes that are responsible for the jerks. These processes remain to be elucidated.

1. Introduction

The temporal variations of the Earth's magnetic field have a wide spectrum which may be roughly divided into higher frequencies due to external sources located in the ionosphere and above in the magnetosphere, and longer periods due to internal sources (the dynamo in the external core). The boundary between external and internal sources was formerly thought to be located at a few years (e.g. 4 years by Currie (1968)). However, periods on the order of or shorter than 1 year were more recently shown to have also an internal part as first noted by Courtillot *et al.* (1978) and Malin and Hodder (1982).

These short periods of internal origin are found in the so-called "geomagnetic impulses" or "geomagnetic jerks". The jerks were expressed in the form of two second degree polynomials of time with a sudden change in curvature at the time of the event; the corresponding secular variation (the first time derivative of the geomagnetic field) is a V-shaped graph, the second time derivative is step-like and the third time derivative is a Dirac distribution. Geomagnetic jerks have been discussed by a number of authors (Courtillot and Le Mouël, 1976, 1984; Courtillot *et al.*, 1978; Malin and Hodder, 1982; Malin *et al.*, 1983; Gubbins, 1984; Kerridge and Barraclough, 1985; McLeod, 1985, 1992; Gavoret *et al.*, 1986; Gubbins and Tomlinson, 1986; Whaler, 1987; Golovkov *et al.*, 1989; Stewart and Whaler, 1992). These former analyses have demonstrated the worldwide character and the internal origin of the events. This internal origin is now generally accepted. However, some authors (Alldredge, 1975, 1984; Shapiro and Akhmetzanova, 1997) observe that the jerk is absent in many observatories and think it possible that some external signal may contribute to enhance the observed changes.

In order to make a systematic study of jerks which have occurred since the beginning of the century, without making any a priori assumption on their existence, location and form, we have recently applied a wavelet analysis to the geomagnetic time series from about one hundred observatories (Alexandrescu *et al.*, 1996). The advantage of this analysis is its special sensitivity to localized events referred to as singularities and defined as discontinuity of some α^{th} derivative of the signal (α, the "regularity" of the singularity, being not necessarily an integer). Seven such events have been detected, two being unquestionably of global extent (1969 and 1978), three being of possibly similar extent (1901, 1913, and 1925), while the remaining two are not seen everywhere at the Earth's surface (1932 and 1949). The two 1969 and 1978 global events (the best documented) display an intriguing spatio-temporal behavior consisting of an early arrival in the Northern hemisphere followed by a later arrival in the Southern hemisphere. In addition, these two events tend to balance each other, as already noted by Stewart (1991) and Le Huy (1995) on the basis of entirely different analysis. The events reveal a more singular behaviour than previously assumed, with a regularity closer to 1.6 than to 2.

In the present study, we will focus on the analysis of the most recent events and compare the last one which has been pointed out in 1991 (Macmillan, 1996; De Michelis *et al.*, 1998) with the 1970 and 1978 impulses. We will be especially interested in the geometry of these jerks. The series are too short after 1991 to perform the wavelet analysis. We will therefore come back to a more classical analysis, as performed by Chau *et al.* (1981), Malin and Hodder (1982) and Gubbins (1984). We will afterwards interpret the geomagnetic jerks in terms of a jump in the acceleration of the fluid motion at the CMB.

2. Data

The data are the observatory annual means (defined as averages over all days of the year and all times of the day)

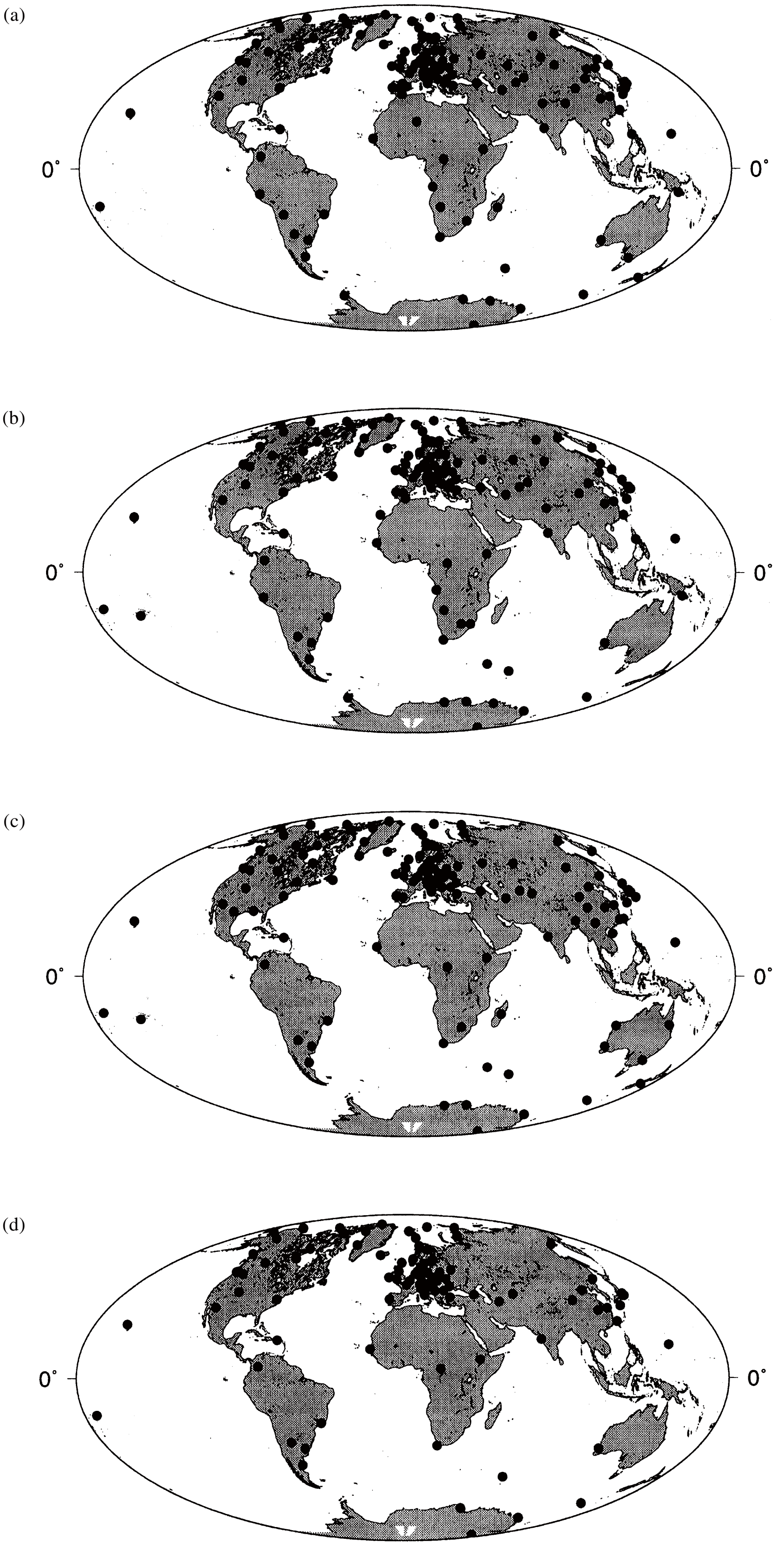

Fig. 1. Distribution of the observatories (a) "dataset 1" used to study the 1969 event; (b) "dataset 2" used to study the 1979 event; (c) "dataset 3" used to study the 1992 event; (d) "dataset 4" with continuous annual mean series from 1960.5 to 1995.5 or 1996.5.

obtained from the British Geological Survey at Edinburgh or directly from the observatories. The three components are X (North magnetic component), Y (East magnetic component) and Z (Vertical downward component).

All time series have been carefully examined; whenever anomalies were detected, additional information was asked from observatories. For some observatories changes in the base-lines had to be applied at certain epochs (for details on this data examination see Alexandrescu, 1996; Alexandrescu *et al.*, 1996).

This study bears on the 1960–1996 time-span. Unfortunately, all the geomagnetic observatories have not sent their data on time, and all the series do not reach 1996. We have finally retained 160 observatories. It is well known that the distribution of magnetic observatories is to be improved, and efforts are presently made in this direction (e.g. INTERMAGNET program). Europe, for instance, is overrepresented. Fortunately, the coefficients of the low degree spherical harmonic analysis we rely on in the following, are not critically affected by this oversampling. More unfortunate is the fact that only 89 observatories out of the 160 have provided continuous annual mean series from 1960.5 up to 1995.5 or 1996.5 (Fig. 1). This forced us to rely on different datasets (described in the next section) depending on the epoch of the jerks to be analysed.

3. Modeling Geomagnetic Jerks

3.1 Date of jerks

The X, Y and Z series are smoothed using the formula:

$$\overline{E}_k = \frac{1}{4}\left(E_{k+1} + 2E_k + E_{k-1}\right)$$

where E_k is the annual mean value of the E component for year t_k. Only the smoothed annual mean values have been used in this study, and the bar above $\overline{E}_k$ will now be deleted.

The secular variation (at January 1st) of year t_k is estimated (in nT/yr) as the approximate temporal derivative

$$\dot{E}_k = \left(\frac{\partial E}{\partial t}\right)_{t=t_k} = E_k - E_{k-1}. \quad (1)$$

The so-computed secular variation of the Y component observed in the 37 observatories located in the Europe is shown on Fig. 2. Three geomagnetic slope changes are clearly visible during the considered period around 1970, 1978 and 1991; two of them have been studied by many authors (Courtillot and Le Mouël, 1976; Courtillot *et al.*, 1978; Kerridge and Barraclough, 1985; McLeod, 1992; Stewart and Whaler, 1992; Alexandrescu *et al.*, 1995, 1996) and the last one has been pointed out by Macmillan (1996) and De Michelis *et al.* (1998).

At this step of the analysis, data are checked again. Indeed, the effect of poorly controlled base-lines appears on the graphs of the secular variation; for some observatories, the uncontrolled base-lines changes appear and produce erratic fluctuations. Finally, we have selected 123 observatories to study the 1969 jerk ("dataset 1"), 124 to study the 1979 jerk ("dataset 2") and other 124 to study the 1992 ones ("dataset 3"). The set of data corresponding to the 89 observatories which have continuously provided annual means (1960.5–1995.5 or 1996.5) will be called "dataset 4". These datasets are shown on Fig. 1.

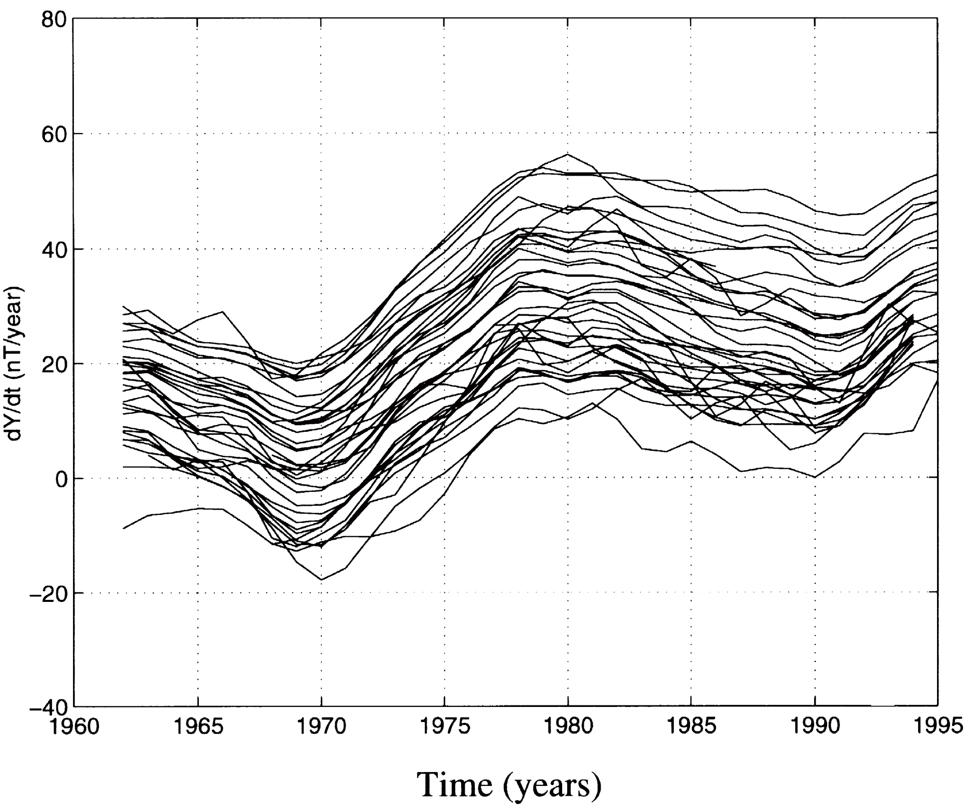

Fig. 2. The secular variation of the Y component observed in the 37 observatories located in Europe.

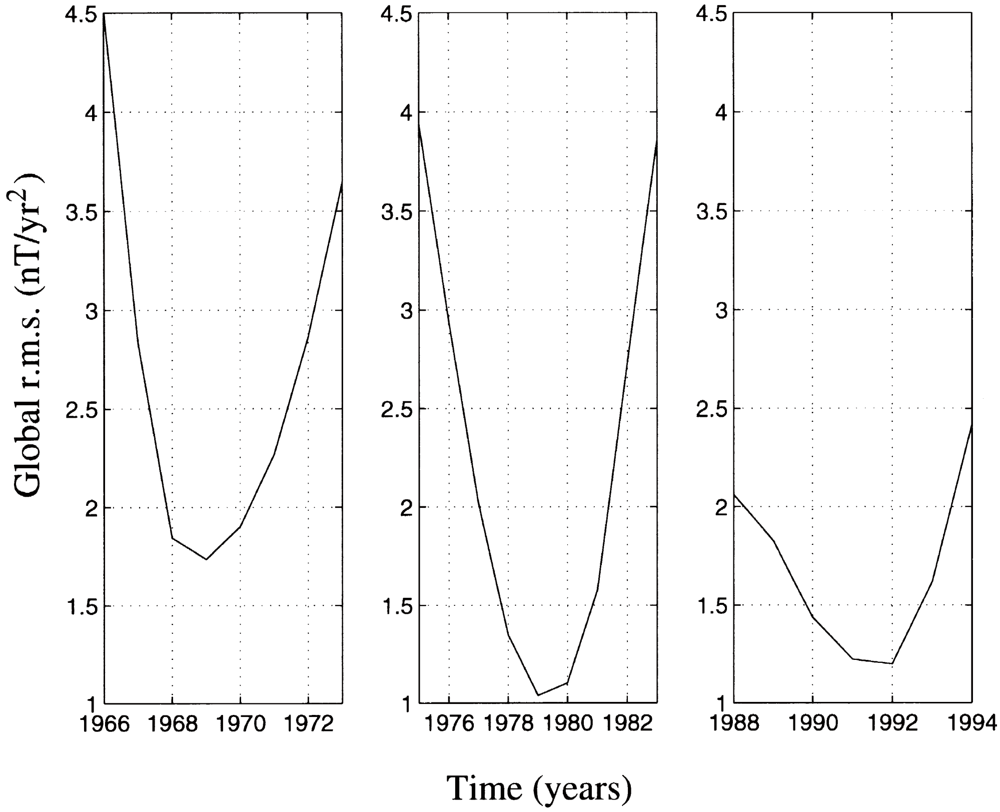

Fig. 3. Variation of the global r.m.s. residual of the secular variation of the Y component around the jerk times. The time for which the global r.m.s. residual is minimum is retained as the date t_0 of the jerk: t_{01} = 1969, t_{02} = 1979 and t_{03} = 1992.

We assume that in a given time interval [t_1, t_2], only one geomagnetic jerk exists at time t_0. To better determine this date we use the method proposed by Le Mouël *et al.* (1982). For a given t_0 we compute for each observatory and for each component (X, Y, Z) the two straight-line segments which best fit the data in the least-squares sense for the intervals before and after t_0:

$$\dot{E}(t) = a_1 t + b_1, \quad t \le t_0,$$
$$\dot{E}(t) = a_2 t + b_2, \quad t \ge t_0. \tag{2}$$

As it is well-known that monthly and annual mean values of the Y component of the geomagnetic field is hardly influenced by the external variations, we first compute t_0 from Y and thereafter assume that this value is also valid for the two other components X and Z. We also assume here that each of the three events takes place simultaneously in all observatories at time t_0.

To derive the best estimate for t_0 we compute the global r.m.s. residual over the period [t_1, t_2] between the observations at each location and the corresponding best two straight-line segments V-shaped curve, for t_0 varying around one of the three above mentioned dates (Le Huy, 1995). The time for which the global r.m.s. residual is minimum is retained as the date t_0 of the jerk (Fig. 3). We first choose t_1 = 1960 and t_2 = 1978 to search t_{01} around 1970. We then take t_1 = 1978 and t_2 = 1996 to search t_{03} around 1991. Next, we take $t_1 = t_{01}$ and $t_2 = t_{03}$ to find t_{02} around 1978. The procedure is then applied again by setting $t_2 = t_{02}$ to search for the final t_{01} and t_{03}. Finally, these t_{01} and t_{03} are used to define a final t_{02}. This led to t_{01} = 1969, t_{02} = 1979 and t_{03} = 1992.

The amplitude of the geomagnetic jerk $\delta\ddot{E}$, for each component, at each observatory, is defined as the difference between the coefficients a_2 and a_1 (Eq. (2)). The values we obtain are of the order of a few nT/yr^2. For the last two jerks, Fredericksburg gives the largest values on X component (–12.3 nT/yr^2 for 1979 and +7 nT/yr^2 for 1992), Fort Churchill gives the largest values on Y component (–8.0 nT/yr^2 for 1979 and +7.2 nT/yr^2 for 1992) and two Canadian observatories give the largest values on Z (at Baker Lake –18.0 nT/yr^2 for 1979 and at Ottawa +13.2 nT/yr^2 for 1992).

The three components $\delta\ddot{X}$, $\delta\ddot{Y}$ and $\delta\ddot{Z}$, for the three jerks, are then submitted to a spherical harmonic analysis.

3.2 Spherical harmonic analysis

The $\delta\ddot{X}$, $\delta\ddot{Y}$, $\delta\ddot{Z}$ are the components of the gradient of the potential $\delta\ddot{V}$:

$$\delta\ddot{V} = a\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left[\left(\frac{a}{r}\right)^{n+1}\left(\delta\ddot{g}_n^m \cos m\phi + \delta\ddot{h}_n^m \sin m\phi\right) + \left(\frac{r}{a}\right)^{n}\left(\delta\ddot{q}_n^m \cos m\phi + \delta\ddot{s}_n^m \sin m\phi\right)\right]P_n^m(\cos\theta) \tag{3}$$

where a is the mean radius of the Earth, and $\delta\ddot{g}_n^m$, $\delta\ddot{h}_n^m$, $\delta\ddot{q}_n^m$ and $\delta\ddot{s}_n^m$ are in nT/yr^2.

The $\delta\ddot{g}_n^m$, $\delta\ddot{h}_n^m$, $\delta\ddot{q}_n^m$ and $\delta\ddot{s}_n^m$ are computed using the classical least squares technique, with the addition of weights given to the data $\delta\ddot{X}(O_i)$, $\delta\ddot{Y}(O_i)$ and $\delta\ddot{Z}(O_i)$ (O_i is the i^{th} observatory). A larger weight is given to an observatory O_i when the two straight-line segments approximation better represent the data (Le Huy, 1995).

The truncation degrees of Eq. (3) have been taken as $N_i = N_e = N = 4$ for internal and external parts (see Malin and Hodder, 1982; McLeod, 1985). Results are given in Table 1.

By analogy with the spatial spectra defined by Lowes (1966) for the main field, we define the spatial spectra of the internal origin (R_i) and external origin (R_e) parts of $\vec{\nabla}\delta\ddot{V}$ as:

Table 1. Spherical harmonic coefficients of 1969, 1979 and 1992 geomagnetic jerks, in nT/yr^2.

Int. coeff.	1969	1979	1992	Ext. coeff.	1969	1979	1992
$\delta\ddot{g}_1^0$	–1.13	0.20	–0.67	$\delta\ddot{q}_1^0$	–0.25	–0.12	–0.47
$\delta\ddot{g}_1^1$	–0.01	–0.31	0.13	$\delta\ddot{q}_1^1$	0.65	–0.01	–1.09
$\delta\ddot{h}_1^1$	0.59	0.27	–0.43	$\delta\ddot{s}_1^1$	–0.04	0.39	–0.51
$\delta\ddot{g}_2^0$	0.90	–0.22	–0.07	$\delta\ddot{q}_2^0$	0.55	0.03	–0.41
$\delta\ddot{g}_2^1$	0.25	0.06	–0.03	$\delta\ddot{q}_2^1$	0.31	–0.25	0.68
$\delta\ddot{h}_2^1$	–2.43	1.16	–1.65	$\delta\ddot{s}_2^1$	–0.31	–0.08	–0.55
$\delta\ddot{g}_2^2$	–0.09	–0.62	0.32	$\delta\ddot{q}_2^2$	0.31	–0.26	0.32
$\delta\ddot{h}_2^2$	–0.58	1.87	–0.11	$\delta\ddot{s}_2^2$	–0.58	0.08	0.37
$\delta\ddot{g}_3^0$	0.90	–0.60	0.29	$\delta\ddot{q}_3^0$	0.15	0.00	–0.02
$\delta\ddot{g}_3^1$	0.69	–0.76	0.29	$\delta\ddot{q}_3^1$	0.02	0.20	0.11
$\delta\ddot{h}_3^1$	–0.69	0.64	0.11	$\delta\ddot{s}_3^1$	0.16	0.25	0.37
$\delta\ddot{g}_3^2$	0.84	0.12	–0.73	$\delta\ddot{q}_3^2$	–0.12	–0.04	0.03
$\delta\ddot{h}_3^2$	–0.27	–0.07	–0.28	$\delta\ddot{s}_3^2$	–0.14	0.41	–0.61
$\delta\ddot{g}_3^3$	0.62	–1.22	1.31	$\delta\ddot{q}_3^3$	–0.31	0.18	0.28
$\delta\ddot{h}_3^3$	–0.42	–0.19	–0.95	$\delta\ddot{s}_3^3$	0.09	–0.05	0.69
$\delta\ddot{g}_4^0$	–0.04	0.25	–0.40	$\delta\ddot{q}_4^0$	–0.06	–0.01	0.26
$\delta\ddot{g}_4^1$	0.13	0.05	–0.43	$\delta\ddot{q}_4^1$	–0.42	0.12	–0.20
$\delta\ddot{h}_4^1$	0.10	–0.01	0.36	$\delta\ddot{s}_4^1$	0.54	–0.29	–0.03
$\delta\ddot{g}_4^2$	–0.60	0.50	0.56	$\delta\ddot{q}_4^2$	–0.08	–0.09	–0.03
$\delta\ddot{h}_4^2$	0.07	0.28	–0.19	$\delta\ddot{s}_4^2$	0.11	–0.20	0.28
$\delta\ddot{g}_4^3$	0.13	0.68	–0.14	$\delta\ddot{q}_4^3$	–0.17	0.05	–0.15
$\delta\ddot{h}_4^3$	0.52	–0.51	0.73	$\delta\ddot{s}_4^3$	–0.06	0.21	–0.17
$\delta\ddot{g}_4^4$	–0.21	–0.04	0.04	$\delta\ddot{q}_4^4$	–0.29	0.15	–0.04
$\delta\ddot{h}_4^4$	–0.62	–0.06	0.86	$\delta\ddot{s}_4^4$	0.07	–0.31	0.02

Table 2. Energy spectrum of the 1969, 1979 and 1992 geomagnetic jerks in (nT/yr^2)^2.

n	$R_i^{1969}(n)$	$R_e^{1969}(n)$	$R_i^{1979}(n)$	$R_e^{1979}(n)$	$R_i^{1992}(n)$	$R_e^{1992}(n)$
1	3.23	0.49	0.41	0.16	1.32	1.68
2	21.42	1.84	15.87	0.29	8.56	2.37
3	12.39	0.56	11.58	0.92	14.13	3.23
4	5.49	2.48	5.64	1.23	10.60	0.94
Total	42.53	5.36	33.51	2.60	34.62	8.23

Table 3. Energy spectrum of the 1969, 1979 and 1992 geomagnetic jerks calculated from the 89 observatories, in (nT/yr^2)^2.

n	$R_i^{1969}(n)$	$R_e^{1969}(n)$	$R_i^{1979}(n)$	$R_e^{1979}(n)$	$R_i^{1992}(n)$	$R_e^{1992}(n)$
1	4.41	1.40	2.57	0.52	1.14	0.74
2	20.22	4.10	14.84	0.30	15.06	6.22
3	19.60	1.94	20.49	4.45	15.75	6.91
4	9.53	3.47	7.19	2.20	6.83	2.68
Total	53.74	10.91	45.08	7.47	38.77	16.56

$$R_i = \frac{1}{4\pi a^2} \iint_{r=a} \left(\vec{\nabla}\delta\ddot{V}_i\right)^2 dS$$
$$= \sum_{n=1}^{N} R_i(n) = \sum_{n=1}^{N} \left[(n+1) \sum_{m=0}^{n} \left[\left(\delta\ddot{g}_n^m\right)^2 + \left(\delta\ddot{h}_n^m\right)^2 \right] \right],$$

$$R_e = \frac{1}{4\pi a^2} \iint_{r=a} \left(\vec{\nabla}\delta\ddot{V}_e\right)^2 dS$$
$$= \sum_{n=1}^{N} R_e(n) = \sum_{n=1}^{N} \left[n \sum_{m=0}^{n} \left[\left(\delta\ddot{q}_n^m\right)^2 + \left(\delta\ddot{s}_n^m\right)^2 \right] \right].$$

Values of $R_i(n)$ and $R_e(n)$ are given in Table 2. The largest part of the jerk is clearly of internal origin: R_i/R_e ratios are respectively 7.9, 12.9 and 4.2 for the 1969, 1979 and 1992 jerks. The energy spectrum of the internal part is dominated by the second and third degrees (but see later). These results are consistent with those obtained by Malin and Hodder (1982) and McLeod (1985) for the 1969 geomagnetic jerk. The $R_e(n)$ values are much smaller than the $R_i(n)$, as already said, and randomly distributed. The $\delta\ddot{q}_n^m$ and $\delta\ddot{s}_n^m$ can be considered as noise. The internal/external ratio is smaller for 1992; this is probably due to the fact that in most observatories the time series are too short after 1992.

In order to estimate the influence of the data distribution, we have also computed models using the 89 observatories of dataset 4 providing time series of identical lengths (Fig. 1). The corresponding energy spectra of the three jerks are presented in Table 3. These spectra are somewhat different from those given in Table 2. But the overall orders of magnitudes and trends clearly remain the same.

Figures 4, 5, and 6 display the maps of the three components of the 1969, 1979 and 1992 jerk fields. Keeping in mind that these maps suffer large uncertainties due to the poor distribution of observatories, especially in the Southern hemisphere, we nevertheless note that the amplitudes of the three events are grossly the same, and that the signs of the different components tend to change from a jerk to the following one, giving an opposite pattern. The change of sign of two successive jerk was pointed out by Golovkov *et al.* (1989) for the "1947", "1958" and "1969" jerks (note that the first two events are not of worldwide extension (Alexandrescu *et al.*, 1996)). Stewart (1991) and Alexandrescu *et al.* (1996) noted the change of sign between the 1969 and 1979 ones.

In order to estimate more quantitatively the correlation between two successive jerks, we have computed the global correlation coefficient (McLeod, 1985):

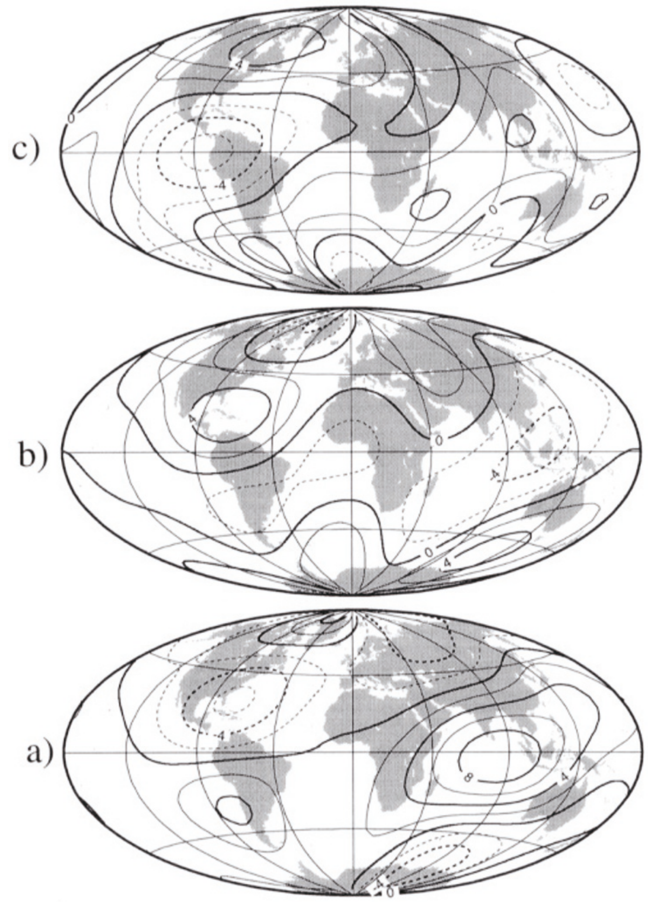

Fig. 4. North component ($\delta\ddot{X}$) of the 1969 (bottom), 1979 (middle) and 1992 (top) jerks. Contour interval: 2 nT/yr^2.

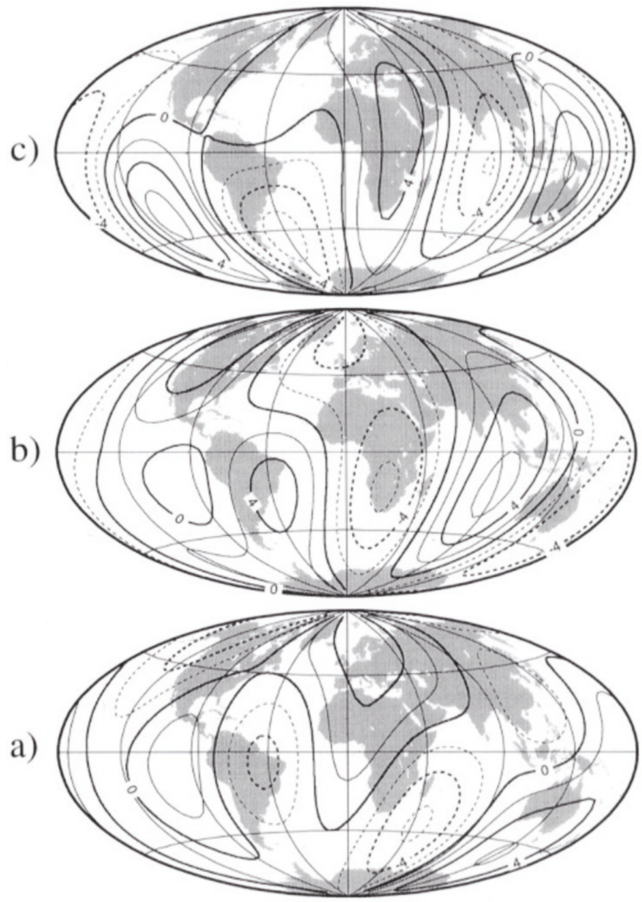

Fig. 5. East component ($\delta\ddot{Y}$) of the 1969 (bottom), 1979 (middle) and 1992 (top) jerks. Contour interval: 2 nT/yr^2.

$$c_{12} = \frac{\sum_{n=1}^{N} (n+1) \sum_{m=0}^{n} \left(\delta\ddot{g}_{1n}^{m} \cdot \delta\ddot{g}_{2n}^{m} + \delta\ddot{h}_{1n}^{m} \cdot \delta\ddot{h}_{2n}^{m} \right)}{\left\{ \left[\sum_{n=1}^{N} (n+1) \sum_{m=0}^{n} \left[\left(\delta\ddot{g}_{1n}^{m} \right)^2 + \left(\delta\ddot{h}_{1n}^{m} \right)^2 \right] \right] \left[\sum_{n=1}^{N} (n+1) \sum_{m=0}^{n} \left[\left(\delta\ddot{g}_{2n}^{m} \right)^2 + \left(\delta\ddot{h}_{2n}^{m} \right)^2 \right] \right] \right\}^{1/2}}. \tag{4}$$

When used up to degree 4, this leads to a correlation coefficient $c_{12} = -0.61$ between 1969 and 1979, of $c_{12} = -0.43$ between 1979 and 1992 and of $c_{12} = 0.31$ between 1969 and 1992. These results clearly confirm the fact that two successive jerks tend to be anti-correlated, the general geometry of the jerks remaining the same. It should also be mentioned that the model for the 1969 jerk we obtain in this study is consistent with those already published by other authors. For example, the correlations between our model and those published by Gubbins (1984) and McLeod (1985) are of 0.89 and 0.91, respectively, when formula (4) is used up to degree 4.

We next want to understand how these results translate in terms of fluid motions at the CMB (Hulot *et al.*, 1993). In what follows only the models derived from the datasets 1, 2, 3 are used (dataset 4 being significantly smaller, models based on it are necessarily less accurate).

c)

b)

a)

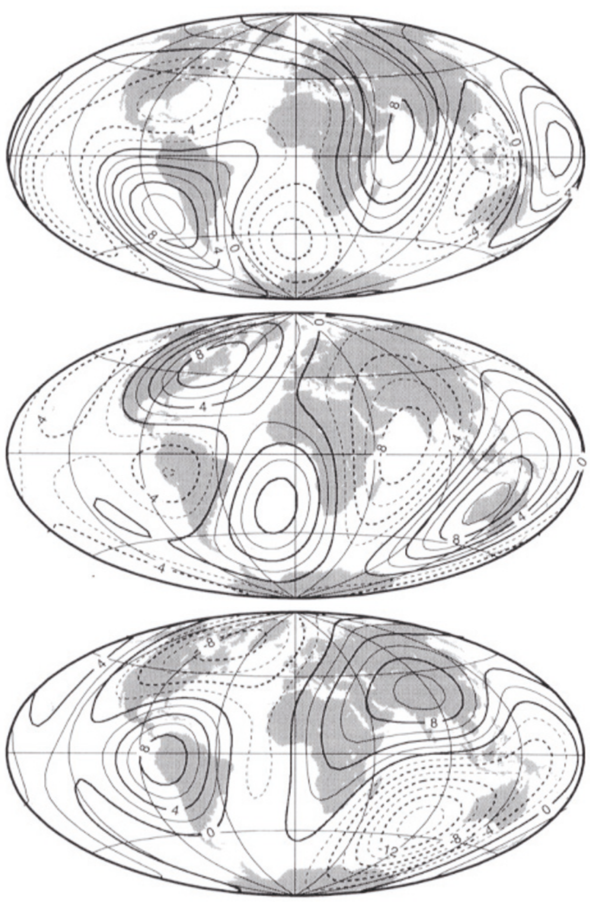

Fig. 6. Vertical component ($\delta\ddot{Z}$) of the 1969 (bottom), 1979 (middle) and 1992 (top) jerks. Contour interval: 2 nT/yr^2.

4. Jerks and Core Flow

For the short time scales considered here, it is legitimate to adopt the frozen flux approximation. The time variation of the radial component B_r of the field at the CMB is then given by the well-known equation (Roberts and Scott, 1965; Gubbins and Roberts, 1987):

$$\dot{B}_r = -\vec{\nabla}_H \cdot \left(\vec{u} B_r \right). \tag{5}$$

$\vec{u}$ is the horizontal velocity field at the CMB, $\vec{\nabla}_H$ the divergence reduced to the horizontal coordinates.

Taking the time derivative of this equation before and after the jerk occurrence time t_0, leads to:

$$\ddot{B}_r^- = -\vec{\nabla}_H \cdot \left(\dot{\vec{u}}^- B_r^- \right) - \vec{\nabla}_H \cdot \left(\vec{u}^- \dot{B}_r^- \right),$$

$$\ddot{B}_r^+ = -\vec{\nabla}_H \cdot \left(\dot{\vec{u}}^+ B_r^+ \right) - \vec{\nabla}_H \cdot \left(\vec{u}^+ \dot{B}_r^+ \right) \tag{6}$$

where "–" and "+" are for the values before and after the jerk. B_r and $\dot{B}_r$ are continuous at $t = t_0$. If we assume that $\vec{u}$ is also continuous (in our model a jerk is a jump in the acceleration of the flow), then:

$$\ddot{B}_r^+ - \ddot{B}_r^- = -\vec{\nabla}_H \cdot \left[\left(\dot{\vec{u}}^+ - \dot{\vec{u}}^- \right) B_r \right],$$

i.e.:

$$\delta\ddot{B}_r = -\vec{\nabla}_H \cdot \left(\delta\vec{\gamma} B_r \right) \tag{7}$$

where $\delta\ddot{B}_r$ is the radial component of the geomagnetic jerk at the CMB (as defined above), and $\delta\vec{\gamma}$ is the jump in the acceleration of the fluid flow at the CMB. B_r and $\delta\ddot{B}_r$ being known, we now wish to compute $\delta\vec{\gamma}$.

Equation (7) is similar to the induction equation (5) that people use to infer core flows (Madden and Le Mouël, 1982). It therefore suffers from the same inability of completely constraining the field $\delta\vec{\gamma}$ to be computed (Backus, 1968). To resolve (at least partly) this ambiguity we adopt the geostrophic approximation (Le Mouël, 1984; Gire and Le Mouël, 1990), which leads to:

$$\vec{\nabla}_H \cdot \left(\delta\vec{\gamma} \cos\theta \right) = 0. \tag{8}$$

The system of Eqs. (7) and (8) then allows the jump $\delta\vec{\gamma}$ at the CMB to be computed if this jump in the acceleration is further known to be large scale (Backus and Le Mouël, 1986). One can represent this jump in the form:

$$\delta\vec{\gamma} = c\vec{\nabla}_H \delta S - c\vec{n} \wedge \vec{\nabla}_H \delta T$$

where c is the core radius, $\vec{n}$ the unit radial outward vector, and δS and δT the corresponding poloidal and toroidal scalar fields. Expanding these fields into spherical harmonics, the acceleration jump can be represented by:

$$\delta\vec{\gamma} = c\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\delta s_n^{mc}\vec{S}_n^{mc} + \delta s_n^{ms}\vec{S}_n^{ms} + \delta t_n^{mc}\vec{T}_n^{mc} + \delta t_n^{ms}\vec{T}_n^{ms}\right) \quad (9)$$

where $\vec{S}_n^{m(c,s)}$ and $\vec{T}_n^{m(c,s)}$ are the elementary poloidal and toroidal vectors (Gire *et al.*, 1986), and $\{\delta s_n{}^m, \delta t_n{}^m\}$ are the poloidal and toroidal coefficients (in rad/yr^2). Within the geostrophic assumption, the acceleration jump field can also be expanded on a basis of elementary geostrophic vectors (Backus and Le Mouël, 1986; Gire and Le Mouël, 1990).

We have applied the inversion method proposed by Gire and Le Mouël (1990) and discussed in Hulot *et al.* (1992) (with an appropriate choice of parameters). The model of B_r is that of Bloxham and Jackson (1992) for the three epochs: 1969, 1979 and 1992 (the model for the last date is extrapolated from the models of the geomagnetic field and secular variation for 1990).

We have also computed the so-called variance reduction of geomagnetic jerks (Bloxham, 1989; Le Huy, 1995):

$$VR(\delta\vec{\gamma}) = \left(1 - \sigma(\delta\vec{\gamma})\right)^2 \cdot 100(\%)$$

where $\sigma(\delta\vec{\gamma})$ is the relative residual between the observed geomagnetic jerk at the Earth's surface $\delta\ddot{\vec{B}}_o$ ($\delta\ddot{X}$, $\delta\ddot{Y}$ and $\delta\ddot{Z}$) and the synthetic jerk $\delta\ddot{\vec{B}}_p$ generated by the acceleration jump $\delta\vec{\gamma}$, through Eq. (7).

$$\sigma(\delta\vec{\gamma}) = \frac{\left\{\iint_{r=a}\left(\delta\ddot{\vec{B}}_p - \delta\ddot{\vec{B}}_o\right)^2 dS\right\}^{1/2}}{\left\{\iint_{r=a}\left(\delta\ddot{\vec{B}}_o\right)^2 dS\right\}^{1/2}}$$

where S is the Earth's surface.

The methods we used to compute $\delta\ddot{\vec{B}}_o$ are obviously rather crude and the observatory distribution is far from being optimum. It would be unreasonable to try and get the small features of the acceleration field at CMB. After some tests we have chosen to truncate Eq. (9) at $N = 4$. Such a choice gives $VR(\delta\vec{\gamma}) \simeq 95\%$ (1969), $VR(\delta\vec{\gamma}) \simeq 91\%$ (1979), $VR(\delta\vec{\gamma}) \simeq 85\%$ (1992) (we have also computed the acceleration jump using the model of the 1969 jerk published by

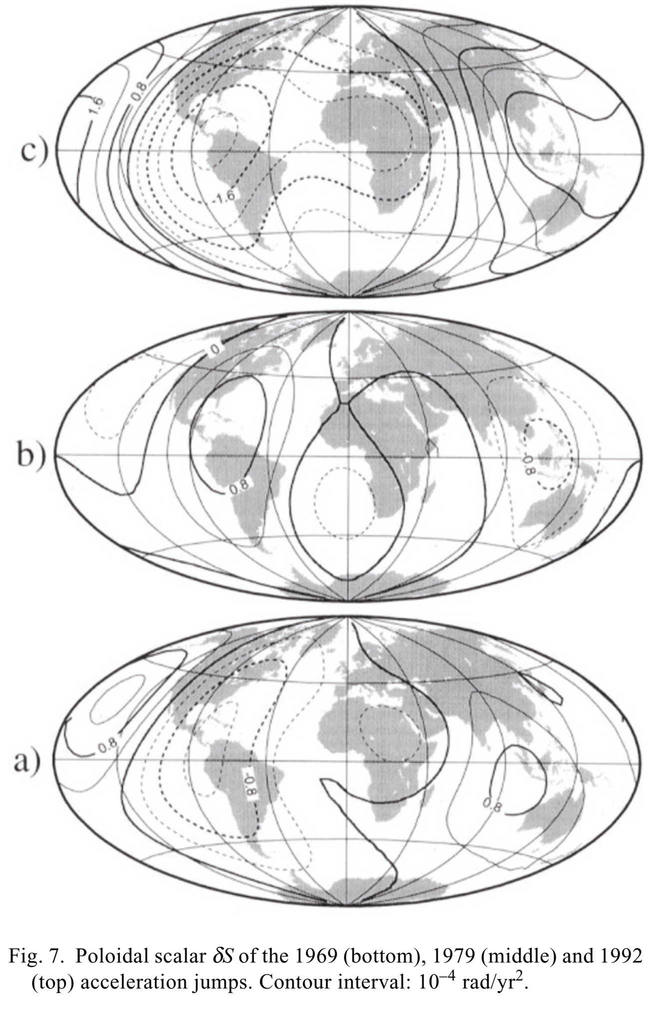

Fig. 7. Poloidal scalar δS of the 1969 (bottom), 1979 (middle) and 1992 (top) acceleration jumps. Contour interval: 10^{-4} rad/yr^2.

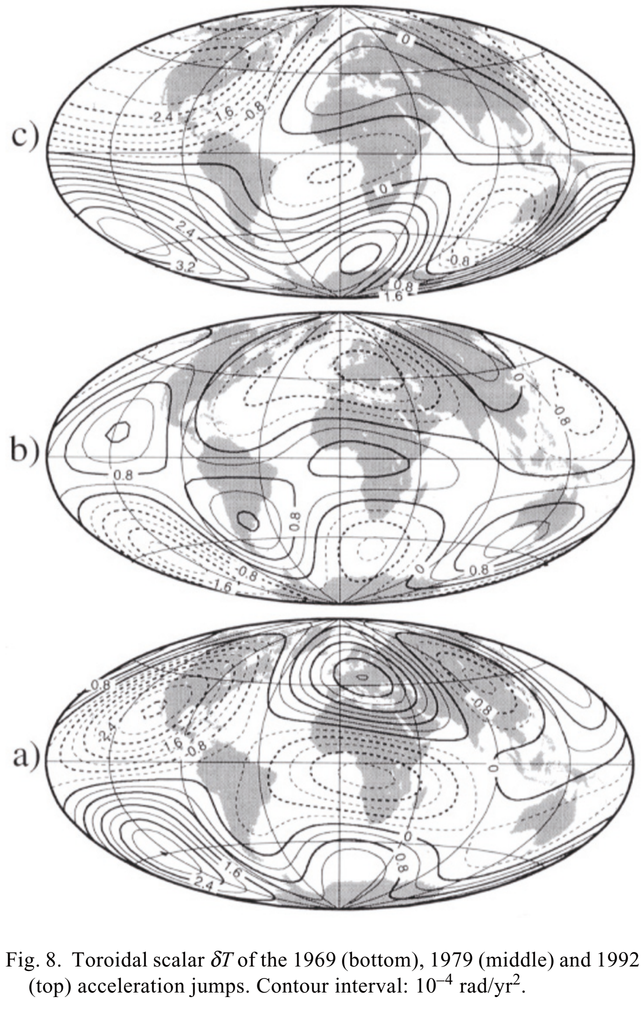

Fig. 8. Toroidal scalar δT of the 1969 (bottom), 1979 (middle) and 1992 (top) acceleration jumps. Contour interval: 10^{-4} rad/yr^2.

Gubbins (1984) and by McLeod (1985), and we obtained $VR(\delta\bar{\gamma}) \simeq 86\%$ and 94%, respectively).

The maps of the three acceleration jumps (of the spherical harmonic expansions of the poloidal and toroidal scalars truncated at $N = 4$) are shown on Figs. 7 and 8. These maps are similar for the three acceleration jumps. More precisely we observe the same global pattern with large scale positive and negative areas on the 1969 and 1992 maps and again on the 1979 map to within a global sign change. The resemblance, although imperfect—and there is little reason to expect it to be perfect—, is certainly striking.

In order to assess this similarity we defined the following correlation coefficient, equivalent to Eq. (4) but applying to the acceleration jumps:

$$k_{12} = \frac{\iint_{\text{core}} \left(\delta\vec{\gamma}_1 \cdot \delta\vec{\gamma}_2\right) dS}{\left[\iint_{\text{core}} \delta\vec{\gamma}_1^2 dS \times \iint_{\text{core}} \delta\vec{\gamma}_2^2 dS\right]^{1/2}}$$
$$= \left[\sum_{n=1}^{N} \frac{(n+1)n}{2n+1} \sum_{m=0}^{n} \left(\delta s_{1n}^{mc} \delta s_{2n}^{mc} + \delta s_{1n}^{ms} \delta s_{2n}^{ms} + \delta t_{1n}^{mc} \delta t_{2n}^{mc} + \delta t_{1n}^{ms} \delta t_{2n}^{ms}\right)\right]$$
$$\cdot \left[\sum_{n=1}^{N} \frac{(n+1)n}{2n+1} \sum_{m=0}^{n} \left[\left(\delta s_{1n}^{mc}\right)^2 + \left(\delta s_{1n}^{ms}\right)^2 + \left(\delta t_{1n}^{mc}\right)^2 + \left(\delta t_{1n}^{ms}\right)^2\right]\right]^{-1/2}$$
$$\cdot \left[\sum_{n=1}^{N} \frac{(n+1)n}{2n+1} \sum_{m=0}^{n} \left[\left(\delta s_{2n}^{mc}\right)^2 + \left(\delta s_{2n}^{ms}\right)^2 + \left(\delta t_{2n}^{mc}\right)^2 + \left(\delta t_{2n}^{ms}\right)^2\right]\right]^{-1/2}. \quad (10)$$

When computed up to degree 4, the correlation coefficient k_{12} is of –0.9 between 1969 and 1979, of –0.6 between 1979 and 1992 and of 0.7 between 1969 and 1992. These values show that the anti-correlation observed between two successive jerks translate into an even more striking anti-correlation between the two corresponding acceleration jumps.

5. Conclusion

The most interesting results of this study are: 1) the similarity between the three acceleration jumps corresponding to the jerks of 1969, 1979, 1992 (with the reservation made for 1992); 2) the change of sign of the acceleration jumps between 1969 and 1979, and between 1979 and 1992; in other words, 1992 comes back to 1969.

Let us now finally speculate about our results. In the simplified model considered above the flow acceleration is constant between two jerks. The consistency of the geometry of the three successive jerks (and of the associated acceleration jumps) together with the changes of sign from one event to the next tend to prevent the flow to step back from some kind of a mean flow and suggest some long time scale memory in the processes that create the jerks. The remarkable and intriguing properties of the jerks evidenced in the present observation-oriented paper, now strongly call for a theoretical interpretation in terms of magneto-hydrodynamics of the upper layers of the core.

Acknowledgments. We are pleased to thank the staff of the geomagnetic observatories for their kind supplying of high-quality data. Dr. S. R. C. Malin and two anonymous referees made very constructive remarks. This is IPGP contribution 1555.

References

Alexandrescu, M., Le champ géomagnétique et ses observatoires, Thèse, Institut de Physique du Globe de Paris, 271 pp., 1996.

Alexandrescu, M., D. Gibert, G. Hulot, J.-L. Le Mouël, and G. Saracco, Detections of geomagnetic jerks using wavelet analysis, *J. Geophys. Res.*, **100**, B7, 12557–12572, 1995.

Alexandrescu, M., D. Gibert, G. Hulot, J.-L. Le Mouël, and G. Saracco, Worldwide wavelet analysis of geomagnetic jerks, *J. Geophys. Res.*, **101**, B10, 21975–21994, 1996.

Alldredge, L. R., A hypothesis for the source of impulses in geomagnetic secular variations, *J. Geophys. Res.*, **80**, 11, 1571–1578, 1975.

Alldredge, L. R., A discussion of impulses and jerks in the geomagnetic field, *J. Geophys. Res.*, **89**, 4403–4412, 1984.

Backus, G. E., Kinematics of geomagnetic secular variation in a perfectly conducting core, *Philos. Trans. R. Soc. Lond., Se. A*, **263**, 239–266, 1968.

Backus, G. E. and J.-L Le Mouël, The region of the core-mantle boundary where a geostrophic velocity fields can be determined from frozen-flux magnetic data, *Geophys. J. R. Astron. Soc.*, **85**, 617–628, 1986.

Bloxham, J., Simple models of fluid flow at the core surface derived from geomagnetic field models, *Geophys. J. R. Astron. Soc.*, **99**, 173–182, 1989.

Bloxham, J. and A. Jackson, Time-dependent mapping of the magnetic field at the core-mantle boundary, *J. Geophys. Res.*, **97**, 19537–19564, 1992.

Chau, H. D., J. Ducruix, and J.-L Le Mouël, Sur le caractère planétaire du saut de variation séculaire de 1969–1970, *C. R. Acad. Sci. Paris*, **293**, Série B, 157–160, 1981.

Courtillot, V. and J.-L. Le Mouël, On the long-period variations of the Earth's magnetic field from 2 months to 20 years, *J. Geophys. Res.*, **81**, 2941–2950, 1976.

Courtillot, V. and J.-L. Le Mouël, Geomagnetic secular variation impulses, *Nature*, **311**, 709–716, 1984.

Courtillot, V., J. Ducruix, and J.-L. Le Mouël, Sur une accélération récente de la variation séculaire du champ magnétique terrestre, *C. R. Acad. Sci. Paris*, **287**, Série D, 1095–1098, 1978.

Currie, R. G., Geomagnetic spectrum of internal origin and lower mantle conductivity, *J. Geophys. Res.*, **73**, 2779–2786, 1968.

De Michelis, P., L. Cafarella, and A. Meloni, Worldwide character of the 1991 jerk, *Geophys. Res. Lett.*, **25**, 377–380, 1998.

Gavoret, J., D. Gibert, M. Menvielle, and J.-L Le Mouël, Long-term variations of the external and internal components of the Earth's magnetic field, *J. Geophys. Res.*, **91**, B5, 4787–4796, 1986.

Gire, C. and J.-L. Le Mouël, Tangentially geostrophic flow at the core-mantle boundary compatible with the observed geomagnetic secular variation: the large-scale component of the flow, *Phys. Earth Planet. Inter.*, **59**, 259–287, 1990.

Gire, C., J.-L. Le Mouël, and T. Madden, Motions at the core surface derived from SV data, *Geophys. J. R. Astron. Soc.*, **84**, 1–29, 1986.

Golovkov, V. P., T. I. Zvereva, and A. O. Simonyan, Common features and differences between "jerks" of 1947, 1958 and 1969, *Geophys. Astrophys. Fluid Dyn.*, **49**, 81–96, 1989.

Gubbins, D., Geomagnetic field analysis, II, Secular variation consistent with a perfectly conducting core, *Geophys. J. R. Astron. Soc.*, **77**, 753–766, 1984.

Gubbins, D. and P. H. Roberts, Magnetohydrodynamics of the Earth's core, in *Geomagnetism 2*, edited by J. A. Jacobs, pp. 1–183, Academic Press, London, 1987.

Gubbins, D. and L. Tomlinson, Secular variation from monthly means from Apia and Amberley magnetic observatories, *Geophys. J. R. Astron. Soc.*, **86**, 603–616, 1986.

Hulot, G., J.-L. Le Mouël, and J. Wahr, Taking into account truncation problems and geomagnetic model accuracy in assessing computed flows at the core-mantle boundary, *Geophys. J. Int.*, **108**, 224–246, 1992.

Hulot, G., M. Le Huy, and J.-L. Le Mouël, Secousses (jerks) de la variation séculaire et mouvements dans le noyau terrestre, *C. R. Acad. Sci. Paris*, **317**, Série II, 333–341, 1993.

Kerridge, D. J. and D. R. Barraclough, Evidence for geomagnetic jerks from 1931 to 1971, *Phys. Earth Planet. Inter.*, **39**, 228–236, 1985.

Le Huy, M., Le champ géomagnétique, les mouvements du fluide à la surface du noyau et les variations décennales de la rotation de la Terre, Thèse, Université de Paris VII, 274 pp., 1995.

Le Mouël, J.-L., Outer-core geostrophic flow and secular variation of Earth's geomagnetic field, *Nature*, **311**, 734–735, 1984.

Le Mouël, J.-L., J. Ducruix, and C. Ha Duyen, The worldwide character of the 1969–1970 impulse of the secular acceleration rate, *Phys. Earth Planet. Inter.*, **28**, 337–350, 1982.

Lowes, F. J., Mean square values on the sphere of spherical harmonic vector fields, *J. Geophys. Res.*, **71**, 2179, 1966.

Macmillan, S., A geomagnetic jerk for the early 1990's, *Earth Planet. Sci. Lett.*, **137**, 189–192, 1996.

Madden, T. and J.-L. Le Mouël, The recent secular variation and the motions at the core surface, *Philos. Trans. R. Soc. Lond.*, **A306**, 271–280, 1982.

Malin, S. R. C. and B. M. Hodder, Was the 1970 geomagnetic jerk of internal or external origin?, *Nature*, **296**, 726–728, 1982.

Malin, S. R. C., B. M. Hodder, and D. R. Barraclough, Geomagnetic variation: a jerk in 1970, in *75th Anniversary Volume of Ebro Observatory*, edited by J. O. Cardus, pp. 239–256, Ebro Obs., Tarragona, Spain, 1983.

McLeod, M. G., On the geomagnetic jerk of 1969, *J. Geophys. Res.*, **90**, 4597–4610, 1985.

McLeod, M. G., Signals and noise in magnetic observatory annual means. Mantle conductivity and jerks, *J. Geophys. Res.*, **97**, 17261–17290, 1992.

Roberts, P. H. and S. Scott, On the analysis of the secular variation. 1. A hydromagnetic constraint: theory, *J. Geomag. Geoelectr.*, **17**, 137–151, 1965.

Shapiro, V. A. and A. A. Akhmetzanova, Jerks in the thin structure of the geomagnetic secular variations in the course of 21st and 22nd solar cycles: their signature and origin, *Abstracts 8th Assembly of IAGA*, session 1.02, Uppsala, 1997.

Stewart, D. N., Geomagnetic impulses and the electrical conductivity of the lower mantle, Ph.D. Thesis, The University of Leeds, Department of Earth Sciences, 191 pp., 1991.

Stewart, D. N. and K. Whaler, Geomagnetic disturbance fields: An analysis of observatory monthly means, *Geophys. J. Int.*, **108**, 215–223, 1992.

Whaler, K. A., A new method for analysing geomagnetic impulses, *Phys. Earth Planet. Inter.*, **48**, 221–240, 1987.

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