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Forecast of the most geomagnetically disturbed days

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Perturbed periods characterized by the AA^* index compiled from 1868 to 1992 are considered. Our analysis of the geomagnetic activity is done by considering the even-odd effect, that was observed in the maximum solar activity and applying Gumbel's first asymptotic distribution extreme values. The present study shows that the largest geomagnetically perturbed days should have different intensities depending on the parity of the solar cycles in which they occur.

1. Introduction

The study of the greatest geomagnetic storms is becoming more and more important. The ring current index Dst, has traditionally been used to identify geomagnetic storms, but other magnetic indices have also been used to characterize larger and the largest storms. According to this, we use a predictive method to evaluate when the next major magnetic storms should occur.

The three-hourly aa global index of geomagnetic activity is calculated by using the magnetograms of two observatories located in England and Australia. The daily average AA^* index is derived from the aa index by considering the maximum average 24-hour global disturbance for each major magnetic storm. It is available for a long span of years and it is compiled by the World Data Center A for STP. Figure 1 shows the histogram for the data set studied. It presents the tendency characteristic of extreme value distributions related to the tail that appears to the right side.

Gumbel's first asymptotic distribution technique (Gumbel, 1954) applied to the largest geomagnetic storm periods in nine solar cycles as characterized by the average half-daily aa indices was considered by Siscoe (1976). Gumbel's third extreme distribution (Gumbel, 1954) to predict the occurrence of largest perturbed periods by using the Dst index was studied by Silbergleit (1996).

In the present paper, we carry out the theory of extreme value statistics applied to magnetic activity by introducing a novel method to select the AA^* peaks. It is based on the evidence that solar activity (for the solar cycles studied) shows an "even-odd" effect (Wilson, 1992 and Letfus, 1994).

Cliver *et al.* (1996) show that a natural 22-year variation is manifested in the low-high interchange of even-odd sunspot maxima within the last six Hale cycles and it is the predominant source of the 22-year cycle in geomagnetic activity. This is the most newly revealed of the major geomagnetic periodicities.

2. The Extreme Value Distributions

For a given maximum AA^* value, the probability that this value be less than a is called $f_n(a, b, u)$. The probability that this value be greater than a, where a represents the extreme value of AA^* in a solar cycle, is estimated by $[1-f_n(a, b, u)]$. The theory of extremes provides the mathematical expression of $f_n(a, b, u)$ with n = I, II, III (Gumbel, 1954).

The first asymptotical function distribution of the extreme value of Gumbel is defined by:

$$f(a, b, u) = \exp[-\exp(-y)] \quad (1)$$

and

$$y = (a-u)/b = A + Ba \quad (2)$$

where

$$u = -A/B \qquad \text{and} \qquad b = 1/B. \quad (3)$$

Considering the characteristics of the data set, the distribution of the extremes will be one of the three following types:
Type I: When the series of data are studied without bounds.
Type II: When the series of data are studied as bounded below.
Type III: When the series of data are studied as bounded above.

For bounded data from below or from above, Gumbel's first asymptotic distribution defined by the Eq. (1) is modified by adding one of these additional mathematical conditions, and thus, Gumbel's second or third asymptotic distributions are obtained.

Largest values of the AA^* index satisfy the condition of being extreme values, because there are approximately 4000 values per solar cycle. Another requisite of the theory is that the values be independent. Thus no more than one value is taken from a single disturbed period. The values of the daily AA^* index of largest independent values for each solar cycle are given in Table 1.

In preparing the AA^* maximum values per solar cycle for plotting, the measurements are ranked in ascending order from 1 to 6, for odd and even cycles separately, as it is shown in Fig. 1. Each observed value is assigned a probability according to O'Connor (1988):

$$Pi = (i-0.3)(N+0.4)^{-1} \quad (4)$$

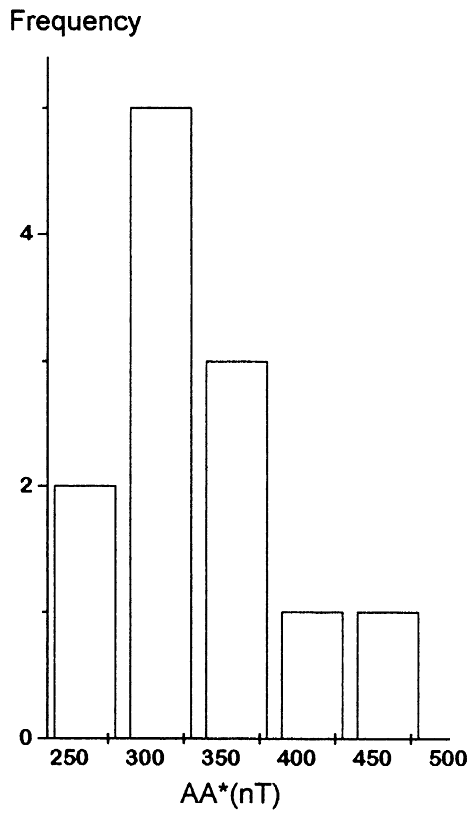

Fig. 1. Histogram of largest average 24-hour global disturbance AA^* index.

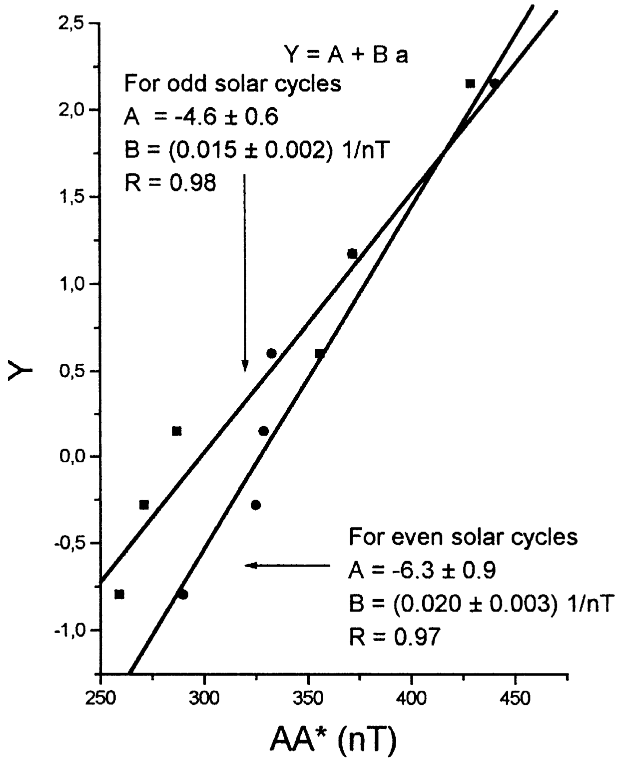

Fig. 2. Events considered to obtain Gumbel's first distribution curve. The probability (abscissa) of each data point (*Pi*) is calculated by Eq. (4). The points corresponding to the largest geomagnetically disturbed periods per odd- and even-numbered cycles are indicated by squares and circles respectively.

Table 1. Maximum AA^* index and plotting values per even- and odd-solar cycles.

Solar cycles	Plotting value	AA^* Max (nT)
11	2.656250E-01	259
13	1.093750E-01	271
21	4.218750E-01	287
20	1.093750E-01	290
16	2.656250E-01	325
18	4.218750E-01	329
14	5.781250E-01	333
15	5.781250E-01	356
19	7.343750E-01	372
12	7.343750E-01	372
17	8.906250E-01	429
22	8.906250E-01	441

where N is the total number of data and i is the ordinal number related to the observed values $(1, 2, \ldots, N)$.

Although AA^* is the preferred index for this study, we note that other indices give somewhat different storm rankings. For example, Table 1 shows that the largest storm in AA^* since solar cycle 11 occurred in solar cycle 22 (in March 1989). In contrast, the same storm ranked only third among extreme values in Ap since 1932, when Ap was established.

The standard statistical parameters of the variable y are obtained from the distribution function (Gumbel, 1954). The mode (u) is obtained graphically as the abscissa value when the ordinate is equal to zero (see Fig. 2).

The return periods $T(a)$ and $T'(a)$ are calculated by using the expressions:

$$T(a) = [1 - f(a, b, u)]^{-1} \tag{5a}$$

and

$$T'(a) = f(a, b, u)^{-1} \tag{5b}$$

where $f(a, b, u)$ is the cumulative probability that a future AA^* value will not exceed the prior value, $T(a)$ (and $T'(a)$) are the expected times required to have one disturbed period with the extreme equal to or exceeding (and less than) a.

The median value, which is the mid-point of the distribution, is estimated by plotting the observations. The relationship between the reduced variate y and the observed one a indicate that the abscissa is equal to the median value when $T'(a) = T(a)$ as it is shown in Fig. 3.

The arithmetic mean (m) is the average of all possible results, and is related to u (Krumbein and Lieblein, 1955 and Jenkinson, 1955) by the expression:

$$m = u + 0.57722b. \tag{6}$$

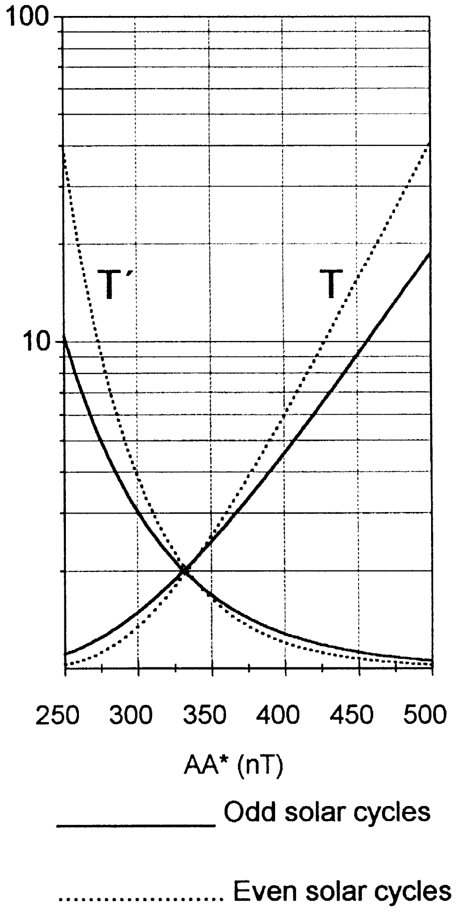

Fig. 3. The curves T (T') show the number of periods required to detect an event with an extreme value equal to or exceeding (less than) a. The plot shows the values related to the largest disturbed days of even and odd solar cycles, separately.

The parameter b is a measure of spread, which is related to the standard deviation (= *SD*) by:

$$SD = b\pi\left(\sqrt{6}\right)^{-1}. \quad (7)$$

3. Event Selection

We studied the temporal distribution of AA^* maximum from 1868 to 1992. On inspecting different indices, it is considered that the hourly (i.e., Dst index) or 3-hr values (i.e., aa index) are not significant when they are used one by one. To obtain more realistic results, the individual daily data (as expressed by the AA^* index) are considered. For each solar cycle, the corresponding AA^* maximum value is considered following two steps:

i) compilation of the AA^* index values calculated from 1868 to 1992 (downloaded at ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC_DATA/INDICES/).

ii) selection of the greatest AA^* values per solar cycle.

The advantages of using the AA^* index are:

a) the availability of extended time records at two old observatories, Greenwich and Melbourne, which are almost antipodal, and

b) the aim of the AA^* series, which provides a daily characterization of the maximum global geomagnetic activity.

4. Results

Figure 2 shows the best fitting adjustments obtained considering Gumbel's first asymptotic distribution, for AA^* values related to even- and odd-numbered solar cycles. According to this and by using Eqs. (2) and (3), the constants are evaluated. They are presented in Fig. 2 and Table 2.

Table 2. Constant values per even- and odd-numbered solar cycles, calculated by considering Eqs. (2) and (3) and Fig. 2.

Cycles	u (nT)	b (nT)
Even	315 ± 92	50 ± 8
Odd	307 ± 80	67 ± 9

The statistical characteristics of the largest AA^* index peaks per solar cycle by considering Eqs. (6) and (7) are $m = 344 \pm 97$ (346 ± 85) nT with $SD = 64$ (86) nT for even- (odd-) numbered solar cycles.

Figure 3 shows the expected number of cycles necessary to have one cycle with the greater magnitude of AA^* less than a, by considering maximum AA^* values per odd and even solar cycles, separately. According to this, for AA^* peak of 400 nT return periods of 5 (or 6) solar cycles for odd (or even) cycles, respectively should be predicted. An AA^* extreme value equal to the largest peak registered should be expected to happen within the next:

i) 8 odd solar cycles or

ii) 13 even solar cycles.

Figure 3 also shows the expansion of the waited rank of extreme values as dependent on the number of cycles. To illustrate it, we consider 10 cycles, the best case being that eight of them would be in the bounded interval defined by:

a) 250 nT and 458 nT for odd cycles or

b) 287 nT and 425 nT for even cycles.

We note that the intensity of the largest disturbed days (for even solar cycles) is waited to be in a narrower interval than the other ones. As the number of periods increases, the indicated ranges also enlarge, Then, the largest disturbed days should happen first in the next odd solar cycles, because for each odd cycle, the range of intensities is greater than the even one. According to this, we could infer that the largest geomagnetically perturbed days do not have the same characteristics in odd as in even solar cycles.

5. Concluding Remarks

In order to predict the occurrence of intense geomagnetic periods (AA^* peaks >250 nT) we took into account the evidence that solar activity (for the solar cycles studied) shows an "even-odd" effect. According to this, we separated the data into two subsets, one of them having the maximum AA^* values observed during the odd cycles and the other one related to AA^* peaks registered during the even-numbered cycles.

By using the present statistical method, we estimated that the next geomagnetic event of intensity equal to the one detected on March 14, 1989 ($AA^* = 441$ nT) should be expected to occur within the next 8 odd or 13 even solar cycles.

According to Fig. 3 we should conclude that the largest geomagnetically perturbed days have different intensities depending on the parity of the solar cycles in which they appear.

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