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Airborne geoid determination

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Airborne geoid mapping techniques may provide the opportunity to improve the geoid over vast areas of the Earth, such as polar areas, tropical jungles and mountainous areas, and provide an accurate "seam-less" geoid model across most coastal regions. Determination of the geoid by airborne methods relies on the development of airborne gravimetry, which in turn is dependent on developments in kinematic GPS. Routine accuracy of airborne gravimetry are now at the 2 mGal level, which may translate into 5–10 cm geoid accuracy on regional scales. The error behaviour of airborne gravimetry is well-suited for geoid determination, with high-frequency survey and downward continuation noise being offset by the low-pass gravity to geoid filtering operation. In the paper the basic principles of airborne geoid determination are outlined, and examples of results of recent airborne gravity and geoid surveys in the North Sea and Greenland are given.

1. Introduction

Precise geoid determination has in recent years been facilitated through the progress in airborne gravimetry. The first large-scale aerogravity experiment was the airborne gravity survey of Greenland 1991–92 (Brozena, 1991). For an extensive review of the potential of airborne gravity for geoid determination see Schwarz (1996).

Airborne gravimetry allows a uniform coverage of gravity to be surveyed in a straightforward fashion, and is especially suited for covering remote and logistically difficult areas such as polar areas, mountains, jungles etc. Another important application is for coastal regions, where airborne gravimetry can "bridge" the gap between satellite altimetry in the open ocean and land gravimetry, and thus improve the geoid determination along the coast. The coastal region has especially been the target of the EU project AGMASCO (Airborne Geoid Mapping System for Coastal Ocanography), for early results see Forsberg (1996).

In this paper we report results on the AGMASCO initial large-scale test carried out in Skagerrak 1996, as well as give results from some more recent KMS measurements in the Arctic Ocean region north of Greenland. AGMASCO surveys have additionally been carried out in the Azores and Fram Strait regions (Bastos *et al.*, 1997). The basic AGMASCO idea is to determine geoid heights (N) from airborne gravity data using standard methods of physical geodesy, and at the same time measure the height above the sea-surface (H) by airborne altimetry. This allows—in principle—the determination of the dynamic sea-surface topography (ζ) through the equation

$$\zeta = h - H - N \qquad (1)$$

where h is the ellipsoidal height of the airplane, determined by kinematic GPS techniques relative to one or more reference receivers on the coast. The determination of ζ is critically dependent on the quality of the geoid heights N.

2. Airborne Geoid Determination Principle

The principle of airborne gravity is quite simple: By flying a modified marine gravimeter the total sum of gravitational and apparent forces are measured, and by using GPS-determined velocity and acceleration results, it is possible to reduce the ficticious forces related to airplane movement. The basic free-air anomaly at altitude is obtained by

$$\Delta g = g - \frac{\partial^2 h_{GPS}}{\partial t^2} + C_{eot} - \gamma_o + \frac{\partial \gamma}{\partial h}(h_{GPS} - N) \qquad (2)$$

where g is measured gravity, h_{GPS} the ellipsoidal height, C_{eot} is the Eotvos correction, γ_o the normal (ellipsoidal) gravity and N the geoid height (an approximate model such as EGM96 is sufficient). The last term of Eq. (2) represents the attenuation of normal gravity with altitude. For high altitude flights second-order terms should be included as well.

It is important for the evaluation of airborne gravimetry that cross-over point analysis is done by free-air anomalies rather than actual gravity, since anomalies to first order will be independent of the actual flight elevation. Due to noise in both gravity and GPS measurements, all quantities entering (2) must be suitably lowpass filtered. In the results of the

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Table 1. Skagerrak airborne gravity cross-over errors and comparion to marine gravimetry.

Unit: mGal	Mean diff.	Standard deviation
Cross-over error before adjustment	—	2.9
do., after bias adjustment	—	1.9
Comparison to new marine data, before adj.	1.0	2.2
do., after adjustment	0.1	2.6

present paper a second-order Butterworth filter was used, with a full-width (half-wavelength) resolution of approx. 100 sec, corresponding to an along-track resolution of approx. 6 km. This filter was used consistently on both gravimetry and GPS data, after conversion of GPS heights to accelerations by a simple double-difference difference equation applied to the 1 Hz data.

The gravity g of (2) is in our case measured by a Lacoste and Romberg S-model gravimeter, a quite complex gravimeter, where both spring tension and beam velocity are used for measuring gravity changes, relative to stationary base readings at the airports. For an in-depth description of the LC&R marine gravimeter see Valliant (1989). The determination of the scale factor relating gravimeter spring beam velocity and the spring tension (the "k"-factor) is done from the airborne data themself, cross-correlating vertical phugoid accelerations measured by GPS gravimeter vertical accelerations (cf. Olesen *et al.*, 1997). The cross-correlation process at the same time allows for estimating possible time offsets between the data streams with an accuracy of a fraction of a second (1 Hz data used in the present processing). We have further applied a correction for gravimeter platform tilt, which may be recovered from a combination of platform horizontal accelerometer output and horizontal GPS accelerations, for details see Olesen *et al.* (1997).

Geoid models are determined from the available airborne and surface gravimetric data using a remove-restore technique, where the anomalous (non-ellipsoidal) gravity potential T (related to the geoid through Bruns' formula, $N = T/\gamma$, where γ is normal gravity) is split into three terms

$$T = T_1 + T_2 + T_3. \tag{3}$$

The first term is given by the spherical harmonic expansion of the geopotential to degree $N = 360$ (EGM96, cf. Lemoine *et al.*), T_2 is an (optional) contribution from the local irregularities of the topography (computed from a digital terrain model using numerical integration techniques), and T_3 the residual gravity field due to subsurface structures. The residual geoid contribution is computed from the residual gravity by Fourier methods

$$N_3 = \Im^{-1}\left\{\Im(S)\Im(\Delta g_3 \sin\phi)\right\} \tag{4}$$

where $\Im$ is the two-dimensional Fourier transform, S the classical Stokes' function (cf. Heiskanen and Moritz, 1967), Δg_3 the reduced gravity anomalies (corresponding to T_3), and ϕ the geographical latitude. Spherical modifications are used, so that the convolution (4) may be formulated virtually exact on the sphere, for details see Forsberg and Sideris (1993).

In the present implementation land and airborne data are gridded using least-squares collocation, taking into account taylored covariance functions of data and the measurement error standard deviations (2 mGal have been assumed uniformly for the airborne data, cf. below).

3. The Skagerrak Airborne Geoid Survey Case

Skagerrak is the approx. 100 km wide strait between Denmark and Norway. It is an area of major oceanographic sea-surface signals, as well as some distinct gravity anomalies due to Tertiary intrusions. Skagerrak has a well-known gravity field from older marine gravity surveys. It was therefore used as a first full-scale survey test area in the AGMASCO project, flown during 9 days in September, 1996, using a Do-228 airplane ("Polar 4") of the Alfred Wegener Institute, Germany. Flights were flown at a nominal height of 1200 ft and speed 130 knots.

GPS solutions for the airplane were computed from reference stations in Denmark, Norway and Sweden using a combination of commerical ("Geotracer") and special developed software (Xu *et al.*, 1994). Sea-surface height was measured with laser altimeter and radar. Comparisons of aircraft vertical accelerations derived from either GPS or laser altimeter generally agreed at the level below 1 mGal, whereas the absolute GPS height errors were found to be 20–30 cm r.m.s. level. It is therefore a challenge to estimate ζ, which has typically a variation of only 10–20 cm. The absolute height errors have, however, only a minor effect on the gravity results.

Table 1 shows the results of a cross-over analysis of all the processed gravity data, using an improved processing scheme compared to the results presented in Olesen *et al.* (1997). The table shows the r.m.s. cross-over errors for 54 cross-over points, both for the original data and for bias-adjusted data, where a unknown bias is estimated for each track, as commonly done in kinematic gravimetry to elimate instrument drifts and systematic errors in Eotvos corrections. Also shown are comparisons to independent high-quality marine survey data, collected by the University of Bergen research vessel "Håkon Mossby" as part of the project. These data have an estimated accuracy around 1 mGal.

From Table 1 it appears that the r.m.s. accuracy of the airborne data is better than 2 mGal (r.m.s. crossing error divided by $\sqrt{2}$).That means that the airborne gravity data is actually better from an r.m.s. point of view than much of the older ship data of the region, which have an estimated standard deviation of 3 mGal (Andersen, 1966). It is also seen from Table 1 that the use of a bias adjustment actually degrades the result, when compared to the independent ship information. No bias term was therefore applied in the final data set, used for geoid computations.

An independent test of data accuracy involves compari-

Table 2. Comparion of geoids to TOPEX altimetry and GPS levelling.

Unit: meter	Mean	Standard deviation
Topex: Geoid from surface data only	0.533	0.157
Topex: Geoid from airborne data	0.511	0.159
GPS-levelling: Geoid from surface data	0.047	0.027
GPS levelling: Geoid from airborne data	0.039	0.027

Table 3. Greenland airborne gravity cross-over errors and comparion to ice gravimetry.

Unit: mGal	Mean	Standard deviation
Cross-over error before adjustment	—	2.6
do., after bias adjustment	—	2.1
Comparison to sea-ice gravity, before adj.	−0.1	2.4
do., after adjustment	0.6	2.9

son to upward continued ground thruth data. Table 1 shows additionally the results of the comparison between the modern ship data of the Håkon Mossby survey and the airborne data along six common mairne/airborne tracks. The marine gravity data have been upward continued to 1200 ft by FFT methods using all available gravity data. Considering the noise in the ship data and the upward continuaton process, the comparison results support an r.m.s. airborne accuracy estimate around 2 mGal. The mean values show good agreement, with biases at or below 1 mGal (biases before and after adjustment are not identical because the comparison is only done along a subset of the airborne tracks, coincident with the new marine data). The absence of a major bias in the airborne compared to the shipborne gravity data assures a measure of quality for both data sets, and usefullness for geoid computations. Due to the long-wavelength nature of the geoid data biases may tilt and offset computed geoids significantly.

The Skagerrak geoid computation has been done using spherical FFT methods and formal terrain reductions, repeating the computations behind the current Nordic standard geoid NKG-96 (Forsberg *et al.*, 1996). The NKG-96 geoid solution has provided geoid fits better than 10 cm for GPS/levelling lines across Scandinavia, and is thus of very high quality. The difference between this geoid model and a new geoid computed from airborne gravity data (not using existing marine gravity data in Skagerrak) differed at most 15 cm (largest discrepancies confined to areas where the airborne data covered voids in the previous data coverage). Table 2 shows results of comparisons with first-order coastal GPS levelling points in Jutland, and Topex altimetry data in Skagerrak. It is seen that the airborne and existing geoids are consistent, and are of a high quality on land (the quality at sea cannot be judged, as the Topex data includes the variability of ζ).

To get a quantitative estimate of geoid accuracy, not available in the used FFT methods, an error estimate of the N_3-contribution, cf. (3), have been obtained by least-squares collocation. The covariance model of Forsberg (1987), fitted to available gravity data, has been used. Airborne gravimetry was assumed to have a 2 mGal error, and one geoid value has been assumed known (at the base tide gauge in NW Jutland) to 1 cm accuracy. Collocation thus yields the accuary of the *relative* geoid. Results, shown in Fig. 1, confirm the accuracy of the airborne geoid determination to be well below 10 cm.

4. The Greenland Arctic Ocean Shelf Case

A gravity survey of the shelf regions of Greenland is currently being flown to improve the overall geoid of Greenland. The surveys are carried out by KMS in cooperation with University of Bergen and NIMA. The survey setup is based on the AGMASCO hardware, with new, more compact data logging and INS equipment mounted in a Greenlandair Twin-Otter. The 1998 survey covered the Arctic Ocean areas off northern Greenland (Fig. 2), an area with no previous marine gravimetry due to the remote location and permanent sea-ice cover. For details of the survey operations see Forsberg *et al.* (1999).

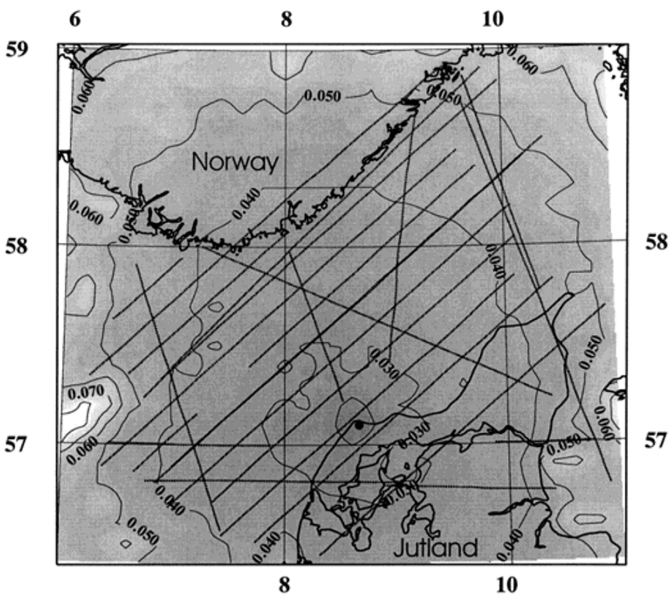

Fig. 1. Flight tracks in Skagerrak and relative collocation geoid error estimates (1 cm contour int.)

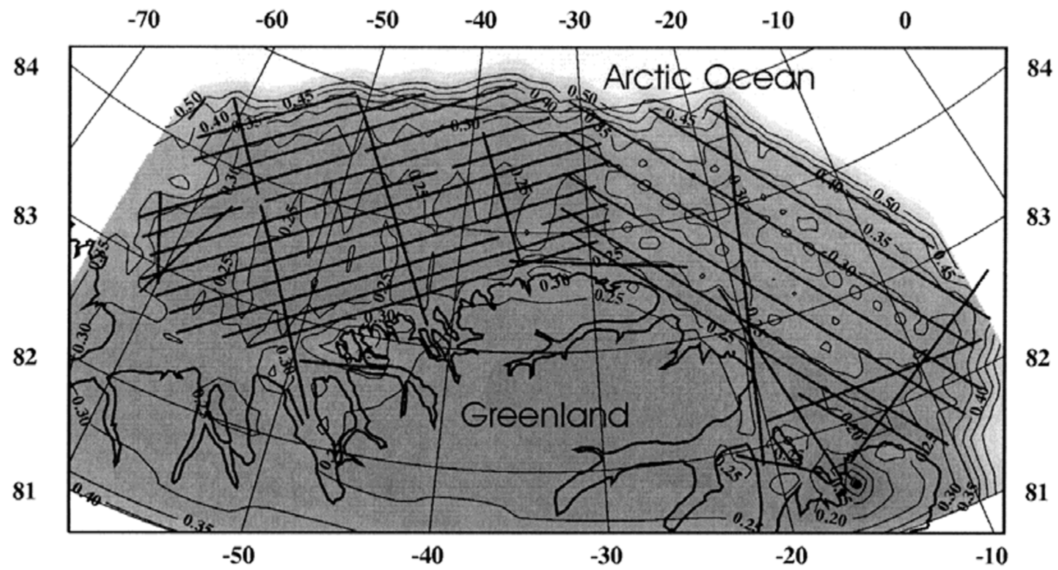

Fig. 2. Airborne gravity tracks north of Greenland and collocation geoid errors (5 cm cont. int.)

The quality of the survey is—in spite of the much longer baselines than in Skagerrak—very satisfactory, with inferred precision around 2 mGal, cf. Table 3, which additionally shows comparison to gravimeter measurements on the ice in the western part of the area.

The geoid of the area was computed analogous to the Skagerrak geoid by spherical FFT methods, using available point gravity data, older 5' airborne grid data over the interior of Greenland and the new 1998 data, assuming errors of 1, 5 and 2 mGal, respectively. The computed geoid showed changes relative to the hitherto best models of more than 2 m, highlighting the impact of airborne gravity data. The results of the collocation error study is shown in Fig. 2. Errors are relative to Station Nord in the south-eastern corner. It is seen that in this case the geoid errrors are bigger than in Skagerrak, with the larger error values in the west a consequence of insufficient data spacing and coverage, especially in the eastern area (more tracks should have been flown).

5. Conclusions

The AGMASCO airborne geoid survey in the Skagerrak has indicated that relative geoid accuracies below 10 cm can be obtained. Over the larger north Greenland shelf region errors of 20–30 cm are estimated, illustrating the need for relatively dense gravity data for estimating geoids over long distances. With the future gravity field satellite missions this situation will improve. For both areas airborne gravimetry has performed excellently at 2 mGal accuracy and a resolution around 6 km.

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