One-dimensional upward continuation of the ground magnetic field disturbance using spherical elementary current systems

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Ionospheric equivalent currents are defined as spherical sheet currents, which reproduce the observed magnetic disturbances below the ionosphere. One way of determining these currents is to place several so called spherical elementary current systems (SECS) in the ionospheric height and to solve an inversion problem for the amplitudes of these systems. In previous studies this method has been applied to two-dimensional data sets, having both latitudinal and longitudinal spatial coverage (2D SECS method). In this paper a one-dimensional variant of this method (1D SECS) is developed. The 1D SECS method can be used even in those situations where the data set is one dimensional, e.g. with one meridionally aligned magnetometer chain. The applicability of the 1D SECS method is tested using both synthetic and real data. It is found that in real situations the errors in the 1D SECS results are 5–10% in current density profiles and $\sim 5\%$ in integrated currents, when compared to the results of the more accurate 2D SECS method.

1. Introduction

The determination of ionospheric equivalent currents from measurements of ground magnetic field disturbances is of primary interest in ionosphere-magnetosphere research (e.g. Untiedt and Baumjohann, 1993). Several different techniques, like spherical or spherical cap harmonic expansions of the disturbance field (Haines, 1985), have been developed over the years. Amm and Viljanen (1999) introduced the spherical elementary current system (2D SECS) method, which is very flexible and well suited for local and global studies. Advantages of this method are that no global wavelength cut-offs are needed (as in harmonic expansions) and the resolution of the analysis grid (density of elementary systems) may vary from place to place according to the amount of available data. A one-dimensional variant of this method (1D SECS) is now presented.

In this context one-dimensionality means that the situation is independent of the azimuthal coordinate of the chosen spherical coordinate system. In this study we use geographical coordinate system (which we assume to be spherical) and the 1D SECS is derived so that several (infinitely many) 2D SECSs of identical amplitude are placed uniformly at a constant latitude at the ionospheric height. The resulting current system (1D SECS) varies only in latitude. The ground magnetic field of such a system is calculated and the ionospheric equivalent currents may be obtained by placing several 1D SECSs of different magnitude at different latitudes, so that their combined ground magnetic field matches the measured field disturbance as closely as possible. In this approach the effect of internal currents is assumed to be negligible. Alternatively the internal equivalent currents could be modelled

by placing a similar 1D SECS system below the ground. In this way the ground disturbance field could be separated into internal and external parts (see Pulkkinen *et al.*, 2003b, for 2D SECS case).

The 1D SECS method should work best in electrojet situations, when strong, mostly east-west aligned currents flow in the ionosphere, for these often are almost one-dimensional. In addition, the 1D SECS method can be used even if there is data from only one meridian magnetometer chain. In the latter case the original 2D SECS method may not work very well, for the regions just outside the magnetometer network may be badly resolved even when using data from two-dimensionally distributed network, see Pulkkinen *et al.* (2003a), and with one-dimensional chains this problem would be more severe. The applicability of the 1D SECS method is tested in different electrojet conditions using synthetic data. Also other 1D upward continuation methods have been developed over the years, e.g. 1D Fourier (Mersmann *et al.*, 1979) and 1D current strip method (Popov *et al.*, 2001), and in this study we compare the results of the 1D SECS and 1D Fourier methods. Real data is also used to investigate the applicability of the 1D methods to situations that in fact are weakly or strongly two-dimensional. The results of the 2D SECS method are used as references in these cases.

2. 2D SECS and Equivalent Currents

Amm (1997) defined two different spherical elementary sheet current systems, one being divergence-free ($\mathbf{J}_{el,df}$) and the other curl-free ($\mathbf{J}_{el,cf}$), see Fig. 1. The curl-free system is associated with field aligned currents (FACs) corresponding to $\nabla \cdot \mathbf{J}_{el,cf}$ that are assumed to flow radially outwards or inwards. Written in a spherical coordinate system (r', θ', ϕ') , with unit vectors $(\hat{\mathbf{e}}_{r'}, \hat{\mathbf{e}}_{\theta'}, \hat{\mathbf{e}}_{\phi'})$, having its pole at the center of

Fig. 1. The divergence- and curl-free elementary current systems. Poles $\theta' = 0$ of these systems may be placed at different points in the ionosphere.

the elementary systems, the currents are

$$\mathbf{J}_{el,cf}(r', \theta') = \frac{I_{0,cf}}{4\pi R_I} \delta(r' - R_I) \cot\left(\frac{\theta'}{2}\right) \hat{\mathbf{e}}_{\theta'} \quad (1)$$

$$\mathbf{J}_{el,df}(r', \theta') = \frac{I_{0,df}}{4\pi R_I} \delta(r' - R_I) \cot\left(\frac{\theta'}{2}\right) \hat{\mathbf{e}}_{\phi'}. \quad (2)$$

Here $I_{0, [cf/df]}$ is the magnitude of the current system and R_I is the radius of the ionosphere, assumed to be a thin layer 100 km above Earth's surface.

Amm (1997) also showed that any ionospheric horizontal current distribution may be uniquely constructed as a superposition of these elementary systems by placing their poles all over the ionosphere. In fact, the elementary systems defined above can be used to expand any continuously differentiable vector field on a sphere.

The curl-free current system $\mathbf{J}_{el,cf}$, with associated FACs, does not produce any magnetic field below the ionosphere, as can be shown using Ampere's law (Fukushima, 1976). Ionospheric equivalent currents, which are spherical sheet currents that produce the same magnetic effects below the ionosphere as the original 3D currents, can be selected to be divergence-free. Consequently the ground magnetic disturbance field may be reproduced by placing several $\mathbf{J}_{el,df}$ systems in the ionosphere above the area of interest and choosing their magnitudes appropriately. This is the 2D SECS method for determining ionospheric equivalent currents, developed by Amm and Viljanen (1999) and further validated by Pulkkinen *et al.* (2003a). In general, the ground magnetic disturbance is caused by both external (ionospheric)

and internal (induced) currents and only the horizontal part of the measured disturbance field can be fitted using just ionospheric equivalent currents. Field separation into internal and external parts may be carried out by placing another equivalent current layer inside the ground and matching all components of the disturbance field (Pulkkinen *et al.*, 2003b).

3. The 1D SECS Method

3.1 Current distribution and magnetic field

In the 2D SECS method poles of the current system $\mathbf{J}_{el,df}$ are placed at different positions (θ_0, ϕ_0) . The unprimed coordinate system (r, θ, ϕ) with unit vectors $(\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$ refers to the geographic spherical coordinates, whereas the pole of the primed system is fixed to the 2D SECS system in question. The 1D variant of the SECS method is obtained by integrating over the longitude ϕ_0 of the 2D SECS pole, so that the resulting current system is independent of longitude and depends only on latitudes θ of the observation point and θ_0 of the poles.

The ionospheric current density of an 1D elementary system is calculated from the integral

$$\mathbf{J}_{1D} = \frac{I_0}{4\pi R_I} \delta(r - R_I) \int_0^{2\pi} \cot\left(\frac{\theta'}{2}\right) \hat{\mathbf{e}}_{\phi'} d\phi_0. \quad (3)$$

Using identities of spherical trigonometry the angle θ' and unit vector $\hat{\mathbf{e}}_{\phi'}$ can be expressed in terms of coordinates θ, θ_0, ϕ_0 and unit vectors $\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi$. Some details of the calcu-

IMAGE Magnetometer Network

Fig. 2. The north-south chain used in this study is marked in black.

lation are given in Appendix A. The result is

$$\mathbf{J}_{1D}(r, \theta, \theta_0) = \frac{I_0}{2R_l} \delta(r - R_l) \hat{\mathbf{e}}_\phi \begin{cases} -\tan\left(\frac{\theta}{2}\right), & \theta < \theta_0 \\ \cot\left(\frac{\theta}{2}\right), & \theta > \theta_0 \end{cases}. \quad (4)$$

The magnetic field caused by the 1D SECS may be obtained by first calculating the vector potential

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}_{1D}(r_1, \theta_1, \theta_0)}{|\mathbf{r} - \mathbf{r}_1|} d\mathbf{r}_1. \quad (5)$$

Integration may be carried out by expanding the denominator in spherical harmonics and the magnetic field is obtained as $\mathbf{B} = \nabla \times \mathbf{A}$. Details of the calculation are given in Appendix B.

The azimuthal component B_ϕ of the magnetic field is zero. The other components can be written using Legendre polynomials P_l^m (with the Condon-Shortley phase convention) as

$$B_r = \frac{\mu_0 I_0}{2r} \sum_{l=1}^{\infty} \left(\frac{r}{R_l}\right)^l P_l(\cos \theta_0) P_l(\cos \theta) \quad (6)$$

$$B_\theta = \frac{\mu_0 I_0}{2r} \sum_{l=1}^{\infty} \left(\frac{r}{R_l}\right)^l \frac{1}{l} P_l(\cos \theta_0) P_l^1(\cos \theta). \quad (7)$$

These results are valid for $r < R_l$. Similar expressions for the case $r > R_l$, needed e.g. in modelling internal equivalent currents, are given in Appendix B. In numerical calculations

Fig. 3. Various original current distributions and those obtained using 1D SECS and Fourier methods. Magnetometer positions are marked with vertical lines.

of the series in Eqs. (6) and (7) several hundred terms (usually about 200) have to be evaluated to get good convergence. This does not affect the computational efficiency of the 1D SECS method, for the series have to be evaluated just once for a given magnetometer and 1D SECS configuration, as explained below.

3.2 Calculation of equivalent currents

The ionospheric equivalent currents are calculated so that several 1D SECSs are placed at different latitudes $\theta_{0,j}$, $j = 1..n$, and their magnitudes are chosen so that the north-component (x -component) of their combined ground magnetic field matches the measured disturbance at latitudes θ_i , $i = 1..m$, as closely as possible. This may be expressed as a matrix equation

$$\mathbf{B}_x = \overline{\overline{M}} \cdot \mathbf{I}_0, \quad (8)$$

where $\mathbf{B}_x$, $\mathbf{I}_0$ are vectors containing the observed x -components at θ_i and the amplitudes of the 1D SECSs at $\theta_{0,j}$, respectively, and $\overline{\overline{M}}$ is a matrix that gives the effect of a current system at $\theta_{0,j}$ to a magnetometer at θ_i .

The matrix $\overline{\overline{M}}$ depends only on the geometry of the situation and it may be calculated using Eq. (7) once the latitudes of the magnetometers and 1D SECSs are specified. In practice the number of magnetometers (m) is usually much smaller than the number of 1D SECSs (n) placed in the ionosphere. As a rule of thumb the spacing of the 1D SECSs should be smaller than 1/2 of the magnetometer spacing and the area where 1D SECSs are placed should exceed the area of the magnetometer network by few degrees. Some fine

tuning is of course required for different magnetometer networks. Thus the inversion problem is highly underdetermined and special methods must be used to solve the unknown $\mathbf{I}_0$ in Eq. (8). One possibility is the singular value decomposition (SVD, e.g. Press *et al.*, 1992, section 2.6).

In SVD the matrix $\overline{\overline{M}}$ is decomposed into $\overline{\overline{M}} = \overline{\overline{U}} \overline{\overline{w}} \overline{\overline{V}}^T$, where $\overline{\overline{U}}$, $\overline{\overline{V}}$ are orthogonal matrices and $\overline{\overline{w}}$ is a diagonal matrix. The diagonal elements w_{kk} of $\overline{\overline{w}}$ are called the singular values of $\overline{\overline{M}}$ and $n - m$ of them are zeros due to the underdetermination of the problem. Also some of the non-zero singular values may be very small (compared to the largest) and they represent the badly behaving part of the solution. The solution is made well behaving by setting all $w_{kk} < \epsilon \cdot \max(|w_{kk}|)$ equal to zero. Here ϵ is an adjustable parameter (in practise $\epsilon = 0.01..0.1$). After this has been done the problem may be solved as

$$\mathbf{I}_0 = \overline{\overline{V}} \text{diag}(\tilde{w}_{kk}) \overline{\overline{U}}^T \mathbf{B}_x, \quad (9)$$

where $\tilde{w}_{kk} = w_{kk}^{-1}$ if $w_{kk} \neq 0$, zero otherwise.

In the above method only the B_x -component is used as input. In principle, by including the internal equivalent currents, also the vertical z -component could be matched. The inversion problem is solved only for the amplitudes of the 1D SECSs. Because each elementary system has global current distribution, given by Eq. (4), there is no need to solve an inversion problem for the positions of the 1D SECSs, if they are chosen reasonably in the first place. It should also be mentioned that in general the unprimed coordinate system

Fig. 4. Locations of the 190°, 210° and 250° MM chain stations. The north-south chain used in this study is marked in black.

need not be the geographical system. Any other spherical coordinate system, e.g. geomagnetic one, could be used.

4. Application Example—Simulated Data

We selected one north-south chain of magnetometers from the IMAGE network¹ as our observation points (see Fig. 2) and simulated magnetic disturbance data for these stations by placing different one-dimensional ($\mathbf{J}_{1D}(\theta) \parallel \hat{\mathbf{e}}_\phi$) current distributions in the ionosphere.

An electrojet having amplitude A , width D and central latitude L is modelled as

$$\mathbf{J}_{1D}(\theta) = A \exp \left[\frac{-(\theta - L)^2}{2D^2} \right] \hat{\mathbf{e}}_\phi. \quad (10)$$

Several electrojets may be superposed and their current distribution is expanded as a superposition of 1D SECS (Eq. (4)) so that the ground magnetic field can be calculated easily. This expansion is in practise identical to the original current distribution, unless it contains step-functions or the like.

B_x at the magnetometers is then used as input for the 1D SECS method and the output current distribution can be directly compared against the original one. We also compared the results of the 1D SECS method with those of the 1D Fourier method in Cartesian coordinate system (Mersmann *et al.*, 1979).

A general description of the 1D Fourier method is: 1) The x -component of the field is expanded as a Fourier series. 2) In upward continuation to height h the expansion coefficients

Fig. 5. Ionospheric equivalent currents along three meridians as given by the 2D SECS method.

Table 1. The percentual errors of the 1D SECS and Fourier results for test cases A–D of Fig. 3, see Eq. (11).

	A	B	C	D
1D SECS	0.5	3.8	5.6	1.9
Fourier	4.8	3.7	10.7	10.8

of wave number k scale as e^{kh} . 3) Inverse Fourier transform gives $B_{x,\text{cont}}$. 4) Equivalent currents, obtained using Ampere's law, are $J_{y,\text{eq}} = \frac{2}{\mu_0} B_{x,\text{cont}}$.

4.1 Test results

Examples of different test current distributions together with those obtained from the 1D SECS and Fourier methods are shown in Fig. 3. Positions of the twelve magnetometers are also indicated with vertical lines. In the 1D SECS method elementary systems were placed uniformly at 40 km (0.35°) intervals in the ionosphere and the ϵ parameter was 0.015.

The calculation area was extended $\sim 5^\circ$ outside the magnetometer chain. In the Fourier method wavelengths smaller than 80 km were ignored and the data was spline interpolated and windowed before the Fourier transform. All these parameters were varied to find a combination that would work well in all situations studied.

In general the results of the 1D SECS method are better than those of the Fourier method, see Table 1. In cases A and D of Fig. 3, where current distributions are relatively smooth and situated well over the IMAGE chain, the 1D SECS results are almost identical to the original distributions. The gap in the magnetometer chain between $70^\circ - 75^\circ$ does not appear to affect the results in these cases. However, with sharper electrojets the spacing of the magnetometers begins to affect the accuracy of the results. This is illustrated in cases B and C, where some deviations from the original distributions can be seen in both methods. Especially the Fourier method has difficulties with the current peaks and in the 1D SECS result there are some oscillations outside the

Fig. 6. LEFT: Snapshot of latitudinal current distribution over IMAGE at 23:25 UT on 27/8/1998 given by the 1D SECS, Fourier and 2D SECS methods. Eastward currents positive. RIGHT: Integrated eastward (+) and westward (-) currents corresponding to the same situation.

Fig. 7. Same as left panel of Fig. 6, but for 13/11/1998.

electrojets. Furthermore, the 1D SECS method seems to be able to reproduce the original currents over the magnetometer area better than the Fourier method in cases where large part of the currents are flowing outside the chain area (results not shown).

As a quantitative estimate for the performance of the 1D methods we calculated the absolute average error of their results, defined as

$$\text{Error} = \left\langle \frac{|J_{1D} - J_{\text{Orig}}|}{\max(|J_{\text{Orig}}|)} \right\rangle_{\text{lat}}. \quad (11)$$

Here $\langle \rangle_{\text{lat}}$ means average over latitude, J_{1D} is the 1D current density (1D SECS or Fourier) and J_{Orig} is the original

current density. Results for the four test cases shown in Fig. 3 are given in Table 1. These results can be compared directly, for $\max(|J_{\text{Orig}}|) = 1 \text{ Am}$ in all test cases.

5. Application Example—Real Data

In order to test the 1D SECS method with real data we selected three different electrojet situations observed with IMAGE and one observed with the 210 Magnetic Meridian chain². The 2D SECS method was used as a reference in the IMAGE cases. The 1D methods used data from only one north-south chain of magnetometers, consisting of 11

Fig. 8. Same as Fig 5, but for the time of panel B of Fig. 7.

Fig. 9. Same as right panel of Fig. 6, but for 13/11/1998.

Fig. 10. Same as left panel of Fig. 6, but for 24/11/2001.

stations when using IMAGE data (Fig. 2, no data from TAR) and of 10 station with 210 MM (Fig. 4). The 2D SECS method used data from all available IMAGE stations.

As the first example we studied a shear flow situation that occurred close to UT midnight on 27/8/1998 over IMAGE. The situation was essentially stationary during the studied time period 23:23–23:37 UT and the two-dimensional equivalent currents obtained from the 2D SECS method are shown in Fig. 5. Both of the two electrojets are tilted from the east-west direction and they are not exactly one-dimensional, as their direction and magnitude change along their flow direction. The latitudinal distribution of east-west aligned equivalent currents crossing the 1D magnetometer chain at 23:25 UT, as given by the 1D SECS, Fourier and 2D SECS methods, is shown in Fig. 6. The total integrated eastward and westward currents for the studied time interval are also shown. The current distribution is similar to test cases C and D of Fig. 3 and both 1D methods give similar results. The 2D SECS method gives slightly different current distribution and integrated currents. These differences are probably caused by the tilt and non-1D nature of the electrojets.

The second example is a substorm that occurred in the evening of 13/11/1998. The results of the different methods are shown in Figs. 7 and 9. The direction of the electrojet varied between southwest and northwest, as determined from the 2D SECS results, and most of the time the magnitude of the current varied with longitude. The greatest differences from one-dimensionality occurred around 22:10 UT, see Fig. 8, but even then the 1D methods gave quite good results, as shown in panel B of Fig. 7. It appears that the non-one-dimensionality of the electrojet affects the 1D methods

more than slight deviations from east-west direction. In the situation of panel D in Fig. 7, at 23:00 UT, the westward electrojet was almost 1D but tilted about 30° from west towards south.

The third example is a local storm on 24/11/2001 that began with a sudden storm commencement (SSC) just before 6 UT and lasted about 2 hours. Results for this case are shown in Figs. 10 and 11. As in the previous cases, the integrated currents given by different methods are very similar to each other, while the latitudinal current profiles show some differences. In general, the current systems were almost all the time 2D and tilted from the east-west direction, sometimes very much so. Situation was worst around 7:50 UT, when large part of the currents were flowing southwards and the current system was quite complex and variable. However, even in this case the 1D methods were able to reproduce the main characteristics of the east-west currents.

The errors of the 1D SECS and Fourier results in the above three examples are listed in Table 2. The errors in the current profiles were calculated as in Eq. (11) for each timestep, except that the 2D SECS results were used as reference. These errors, averaged over time, are given in the upper part of Table 2. In the lower part of Table 2 are the errors in the integrated currents for the three examples. These were calculated as

$$\text{Error} = \left\langle \frac{|(J_{E,1D}^{\text{int}} - J_{W,1D}^{\text{int}}) - (J_{E,2D}^{\text{int}} - J_{W,2D}^{\text{int}})|}{J_{E,2D}^{\text{int}} - J_{W,2D}^{\text{int}}} \right\rangle_{\text{time}} \quad (12)$$

Here $E(W)$ means the positive (negative) integrated currents, 1D stands for the 1D SECS or Fourier methods and

Fig. 11. Same as right panel of Fig. 6, but for 24/11/2001.

Fig. 12. Integrated eastward (+) and westward (−) currents for the whole day 24/11/2001 together with IMAGE based indices IU and IL, corresponding to AU and AL, respectively.

Fig. 13. Snapshots of longitudinal current distributions obtained from 210 MM data for 6/4/2000 by 1D SECS and Fourier methods. Eastward currents positive.

Table 2. The percentual errors of the 1D SECS and Fourier results, as compared to the 2D SECS results, for examples 1–3 (see text). Upper part: error in the current profiles averaged over latitude and timesteps (as in Eq. (11), but J_{Orig} replaced with J_{2D}). Lower part: error in the integrated currents averaged over timesteps (as in Eq. (12)).

	Ex. 1	Ex. 2	Ex. 3
1D SECS	9.4	5.4	6.6
Fourier	11.7	6.4	9.4
1D SECS	6.3	5.1	5.6
Fourier	8.6	3.5	4.9

2D for the 2D SECS method. It should be remembered that the results of the 2D SECS case were used as reference when calculating the errors in Table 2 and consequently these error estimates themselves are bit uncertain. Nevertheless, it can be concluded that the 1D SECS method gives slightly more accurate current profiles, whereas the Fourier method seems to give a little better estimate for the integrated currents, except in the first example. This difference may be caused e.g. by slight oscillations in the Fourier current profiles, which average to zero in the integration. In general the errors in the integrated currents are smaller than in the actual current profiles.

In Fig. 12 we have compared the integrated currents for the whole day 24/11/2001 with IMAGE based estimates of the AL and AU indices, called IL and IU, respectively (Kauristie *et al.*, 1996). It can be seen that the IL and IU indices

give a reasonable estimate of the electrojet currents, but the agreement is not perfect. For example, the largest positive and negative values of the indices are about the same whereas the peak values of the eastward currents are larger than those of the westward currents. The integrated currents in Fig. 12 were calculated without data from BJN (see Fig. 2), as it was not available for the whole day. Nevertheless, the results of the three different methods are almost identical.

In the last example we used data from the 210 degree Magnetic Meridian chain in order to test the applicability of the 1D SECS and Fourier methods in magnetometer chains of different lengths. The 1D methods were recalibrated for this geometrically different situation with synthetic data, i.e. the spacing of the 1D SECSs, the ϵ parameter, the Fourier wavelength cut-off and the amount to extend the calculation area outside the magnetometer chain were adjusted. The synthetic data we used simulated different kinds of equatorial and polar electrojets, in the same manner as in section 4 for the IMAGE network. The ionospheric equivalent currents calculated for the beginning of the April 2000 storm (6–7/4/2000) are shown in Fig. 13. Unfortunately there was no data from ADL so it was replaced by CAN (Fig. 4), which is approximately at the same latitude. The current distributions obtained using different 1D methods are similar in shape, except for the northern border of the analysis area where the 1D SECS method gives much larger westward currents. Comparison with simulated calibration cases (not shown) indicates that most of this northern electrojet is real, although current distributions that extend outside the analysis area may not be reproduced accurately. In general,

Fig. 14. Geometry of the 1D SECS current density calculation.

the 1D SECS results should be more accurate than those of the Fourier method, as expected when using data from long magnetometer chains where effects of spherical geometry are more important.

6. Conclusions

We have presented a new 1D method for determining ionospheric equivalent currents from ground magnetometer measurements. This 1D SECS method is applicable to electrojet situations that closely resemble the 1D geometry of the model and it can be used even if data along only one chain of magnetometers is available, as is the case e.g. with the Greenland magnetometer chain³ and satellite measurements. In contrast to other existing 1D upward continuation methods, e.g. Fourier expansion, the 1D SECS method respects the spherical geometry of the situation. Furthermore, there is no need to specify any kind of global wavelength cut-offs (as in harmonic expansions) and the spacing of 1D SECSs can vary with latitude. Smaller spacing (better resolution) may be used over areas where magnetometers are closer to each other.

Tests using simulated data at the IMAGE magnetometer stations indicate that the 1D SECS method indeed works well in various electrojet situations. Test cases with real data from the IMAGE chain confirm this and also indicate that the 1D SECS method gives reliable results even when the ionospheric currents are not 1D. Especially the integrated currents given by the 1D SECS method are almost identical to the results of the more accurate 2D SECS method. As the 1D SECS method is computationally efficient (IMAGE data for a whole day is processed in less than one minute using a standard PC), it seems that there is no need to use local magnetic indices (e.g. IU and IL) to estimate the electrojet currents, for the currents can be directly calculated from the magnetic

data with little extra effort. As a future work we will use the 1D SECS method (with slight modifications) to estimate the ionospheric currents from satellite measurements of the magnetic disturbance field.

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Appendix A.

In this Appendix we outline the derivation of Eq. (4) for the 1D SECS current density. Our starting point is Eq. (3), in which the angle θ' and unit vector $\hat{\mathbf{e}}_{\phi'}$ have to be expressed in terms of coordinates θ, θ_0, ϕ_0 and unit vectors $\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi$. As stated in the text the pole of the primed system is located at (θ_0, ϕ_0) and the point where we want to evaluate the current density is at (θ, ϕ) . Because of symmetry we may set $\phi = 0$. θ' is the latitude of the point (θ, ϕ) in the primed system.

According to basic trigonometry

$$\cot\left(\frac{\theta'}{2}\right) = \frac{\sin\theta'}{1 - \cos\theta'}. \quad (\text{A.1})$$

Also, it is clear from Fig. 14 that the unit vector $\hat{\mathbf{e}}_{\phi'}$ can be expressed as

$$\hat{\mathbf{e}}_{\phi'} = \hat{\mathbf{e}}_\phi \cos C + \hat{\mathbf{e}}_\theta \sin C. \quad (\text{A.2})$$

Using identities of spherical trigonometry it is easy to show that

$$\sin C = \frac{\sin\theta_0 \sin\phi_0}{\sin\theta'}. \quad (\text{A.3})$$

$$\cos C = \frac{\cos \theta_0 - \cos \theta \cos \theta'}{\sin \theta \sin \theta'} \quad (\text{A.4})$$

$$\cos \theta' = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos \phi_0. \quad (\text{A.5})$$

By inserting the above expressions to the integral in Eq. (3) the $\hat{e}_\theta$ and $\hat{e}_\phi$ parts of the current density $\mathbf{J}_{1D}$ may be calculated. The θ component vanishes, as can be expected because of symmetry, and the ϕ component results in integral

$$2 \int_0^\pi \frac{\cos \theta_0 \sin \theta + \cos \theta \sin \theta_0 \cos \phi_0}{1 - \cos \theta \cos \theta_0 - \sin \theta \sin \theta_0 \cos \phi_0} d\phi_0 \quad (\text{A.6})$$

This can be calculated e.g. by using a formula (Gradshteyn and Ryzhik, 1965, p. 366, equation 3.613.1),

$$\int_0^\pi \frac{\cos nx \, dx}{1 + e \cos x} = \frac{\pi}{\sqrt{1 - e^2}} \left(\frac{\sqrt{1 - e^2} - 1}{e} \right)^n, \quad (\text{A.7})$$

in the special cases $n = 0$ and $n = 1$. The above expression is valid when $e^2 < 1$. This condition does not hold if $\theta = \theta_0$, at which point the current density is discontinuous. The result of the integral in Eq. (A6) can be simplified to

$$2\pi \begin{cases} -\tan\left(\frac{\theta}{2}\right), & \theta < \theta_0 \\ \cot\left(\frac{\theta}{2}\right), & \theta > \theta_0 \end{cases}, \quad (\text{A.8})$$

which gives the result in Eq. (4).

Appendix B.

In this Appendix we outline the derivation of Eqs. (6) and (7) for the 1D SECS magnetic field components. Our starting point is Eq. (5) for the vector potential. We are again allowed to choose the observation point (r, θ, ϕ) so that $\phi = 0$. In this case the unit vector $\hat{e}_{\phi_1}$ can be written in Cartesian coordinate system as

$$\hat{e}_{\phi_1} = -\sin \phi_1 \hat{e}_x + \cos \phi_1 \hat{e}_y. \quad (\text{B.1})$$

Integration over r_1 is trivial because of the δ -function in $\mathbf{J}_{1D}$. The remaining double integral is

$$\mathbf{A} = \frac{\mu_0 I_0 R_I}{8\pi} \int_0^\pi [\cos \theta_1 - \text{sign}(\cos \theta_1 - \cos \theta_0)] * \int_0^{2\pi} \frac{-\sin \phi_1 \hat{e}_x + \cos \phi_1 \hat{e}_y}{|\mathbf{r} - \mathbf{r}_1|} d\phi_1 d\theta_1. \quad (\text{B.2})$$

Clearly, the x component vanishes in the ϕ_1 integration so that $\mathbf{A}$ is directed along the $\hat{e}_y = \hat{e}_\phi$ unit vector. The remaining ϕ_1 integral may be calculated by expanding the denominator in spherical harmonics,

$$\frac{1}{|\mathbf{r} - \mathbf{r}_1|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{1}{2l+1} * \frac{r_{<}^l}{r_{>}^{l+1}} Y_{l,m}^*(\theta_1, \phi_1) Y_{l,m}(\theta, \phi). \quad (\text{B.3})$$

Here we have defined $r_{<} = \min(r, R_I)$, $r_{>} = \max(r, R_I)$. Because $Y_{l,m}^*(\theta_1, \phi_1) \propto e^{-im\phi_1}$ and $\cos \phi_1 = \text{Re}(e^{i\phi_1})$ the only value of m that does produce nonzero integral is $m = 1$.

The remaining task is to calculate the θ_1 integral

$$\int_0^\pi [\cos \theta_1 - \text{sign}(\cos \theta_1 - \cos \theta_0)] P_l^1(\cos \theta_1) d\theta_1. \quad (\text{B.4})$$

Here we have used the Condon-Shortley phase convention in the definition of the associated Legendre polynomials P_l^1 . Using their properties and integrating by parts yields the result of the above integral as $2\delta_{l,0} - 2P_l(\cos \theta_0)$.

Using the above results and writing the remaining spherical harmonics in terms of Legendre polynomials the vector potential can be expressed as

$$\mathbf{A}(\mathbf{r}) = \frac{-\mu_0 I_0 R_I}{2r_{>}} \sum_{l=1}^{\infty} \left(\frac{r_{<}}{r_{>}} \right)^l \frac{1}{l(l+1)} * P_l(\cos \theta_0) P_l^1(\cos \theta) \hat{e}_\phi. \quad (\text{B.5})$$

The magnetic field can be calculated as $\mathbf{B} = \nabla \times \mathbf{A}$. The azimuthal component B_ϕ of the field is zero and in the case $r < R_I$ the other components are given in Eqs. (6) and (7). In the case $r > R_I$ the r - and θ -components are

$$B_r = \frac{\mu_0 I_0 R_I}{2r^2} \sum_{l=1}^{\infty} \left(\frac{R_I}{r} \right)^l P_l(\cos \theta_0) P_l(\cos \theta) \quad (\text{B.6})$$

$$B_\theta = \frac{-\mu_0 I_0 R_I}{2r^2} \sum_{l=1}^{\infty} \left(\frac{R_I}{r} \right)^l \frac{1}{l+1} * P_l(\cos \theta_0) P_l^1(\cos \theta). \quad (\text{B.7})$$

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