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A hybrid method to determine a local geoid model—Case study

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This paper studies the methodology to construct an accurate local/regional geoid model. To achieve the objective an integration of all the information/data available must be performed. A hybrid approach of so-called sequential processing is employed in this study. It involves three steps: the construction of a gravimetric geoid using the well-known remove-restore technique; the least squares fitting of 3-parameter transformation to remove the bias and tilts between the gravimetric geoid and GPS/leveling data; further refinement of the geoid model with GPS/leveling data and other information. The study is focused on the last step of signal extraction. The proposed approach was used in the determination of the Hong Kong geoid model and the Shenzhen geoid model. Their accuracy was evaluated with independent GPS/leveling data. These case studies indicate that the absolute accuracy of a centimeter and relative accuracy of some 1 part per million (ppm) of a local geoid model are achievable if the methodology is carefully designed and all the information/data are fully utilized.
Key words: Hybrid method, local geoid model.

1. Introduction

Determination of the geoid has been one of the main research areas in geodesy for several decades. More and more accurate geo-potential models have been developed. With the development of GPS positioning techniques, a great attention has been paid to the precise determination of local/regional geoids, aiming at replacing the geometric leveling with GPS surveys (e.g., Engelis *et al*., 1985). Several methods have been developed, which can be classified into two basic approaches: the geometric approach and the gravimetric approach.

The geometric approach is to use the known "geoid heights" at some points, which are derived from co-located GPS-determined heights and leveled heights (hereafter called "observed" geoid height), to interpolate the geoid heights at other points. "Geoid height" used in this paper refers to the difference between the WGS-84 ellipsoidal height and the leveled height with respect to a local vertical datum. The interpolation can be done graphically or analytically. Yanalak and Baykal (2001) discuss several methods of analytic interpolation. Usually, a plane or low order polynomial is used to model the geoid (e.g., Featherstone *et al*., 1998). The geometric method has been widely used in engineering projects with an area up to tens kilometers squared. The accuracy of the geoid so established depends on several factors, like the distribution and number of GPS/leveling stations, characteristics of the geoid in the region, the method of interpolation, and the accuracy of GPS/leveling data. The gravimetric approach is to determine a geoid model using gravity measurements. The well-known remove-restore technique is commonly used, which is well documented (e.g., Moritz, 1983; Torge, 2001).

In order to determine a precise local geoid, a full advantage of all types of data/information must be taken in an integrated solution. This paper studies the methodology for such a solution using the data in two areas. One is Hong Kong territories of about 1000 kilometers squared, and the other is Shenzhen of over 2000 kilometers squared in the Southern China. Shenzhen neighbors Hong Kong with different vertical datum. The paper first reviews and discusses the methods for integrating heterogeneous data in geoid determination. Then the results of the two study areas with the proposed method are presented and analyzed.

2. The Methodology

2.1 Integrated approach

The rigorous approach for the integration of heterogeneous data/information is to express different types of information/data (e.g., gravity measurements, coefficients of a geo-potential model, topographical data, and GPS/leveling surveys) as functions of geoid heights and solve for them. However, it involves complicated functional models and stochastic models (weighting of different types of information). Thus, other approaches, though approximate, have been developed for practical uses.

The first kind of integration is to incorporate a geo-potential model and local terrain information into the geometric approach (Doerflinger *et al*., 1997; Yang and Chen, 1999). The approach is similar to the remove-restore technique used in the gravimetric approach. Let the geoid height N be separated into three components N_{GM}, N_I and N_T:

$$N = N_{GM} + N_I + N_T \tag{1}$$

where N_{GM} is the long-wavelength component calculated from a geo-potential model, N_T is the terrain correction calculated from the topography information, e.g., a digital terrain model (DTM), and N_I is the medium wavelength component evaluated by an interpretation technique. Values of N_I at GPS/Leveling stations can be calculated from the observed geoid heights N as

$$N_I = N - N_{GM} - N_T \tag{2}$$

The values N_I at these GPS/leveling stations are then used to interpolate the corresponding values $N_I{}'$ at any other points. After the predicted values $N_I{}'$ are obtained, the geoid heights N' at the predicting points are computed by adding the long-wavelength component $N_{GM}{}'$ and the terrain correction $N_T{}'$ at these predicting points:

$$N' = N_{GM}{}' + N_I{}' + N_T{}'(3) \tag{3}$$

The case study over Hong Kong shows that the approach can significantly improve the accuracy of the geoid compared with the pure geometric method (Yang and Chen, 1999).

The second kind of integration is to transform a gravimetric geoid model for better fitting to the GPS/leveling data. The gravimetric geoid is first constructed using gravity measurements with the remove-restore technique. As there may exist systematic biases between the constructed gravimetric geoid and the GPS/leveling derived geoid, a transformation of 4 parameters (e.g., Fotopoulos *et al.*, 1999; Fotopoulos *et al.*, 2003) or 3 parameters (Yang and Chen, 2001) is then applied to the gravimetric geoid using the observed geoid heights (the resultant geoid is hereafter called the "transformed" gravimetric geoid). Although the biases may come from both kinds of geoid models, the transformation is necessary if the constructed geoid model is mainly used for GPS leveling. This procedure is also used when the accuracy of a gravimetric geoid is to be evaluated by the observed geoid heights (e.g., Fotopoulos *et al.*, 1999; Forsberg *et al.*, 1997). Since this procedure can improve the fitting of a gravimetric geoid with respect to GPS/leveling data, Featherstone *et al.* (1998) refer it as the combined gravimetric-geometric method.

The above two kinds of integration, however, do not take full advantage of all the data/information available. The first one does not use gravity measurements. While the second one, though taking into account the general trend of the separation between the constructed gravimetric geoid and the observed geoid heights, does not fully utilize all the information/data. After the least squares fitting of the constructed gravimetric geoid into the observed geoid heights, the residuals (i.e., the differences between the transformed gravimetric geoid heights and observed geoid heights) may not be completely random. Some useful signals may exist, which should be extracted to further improve the accuracy of the geoid model.

We therefore propose an approach of three-step procedure to be used:

1) Construct a gravimetric geoid using the above-mentioned gravimetric approach. The gravimetric geoid is usually given at regular grid points;

2) Remove the bias and tilts of the gravimetric geoid model with respect to GPS/leveling data by using the least squares fitting, i.e., three-parameter least squares transformation;

3) Signal extraction. This study concentrates on the methods to attract useful signals from the differences between the transformed gravimetric geoid model and the observed geoid heights, which is discussed below.

2.2 The methods for signal extraction

There are several possible methods for signal extraction to refine the gravimetric geoid. A popular approach is the least square collocation (LSC). The LSC separates the difference dN_i between the transformed gravimetric geoid height and the observed geoid height at a point into signal and noise parts. The signals at the grid points can be predicted from all the dN values through a covariance function. The predicted signals are then added to the transformed gravimetric geoid heights, resulting in a refined geoid model. Let $d\boldsymbol{N}$ be vector of the differences at all GPS/leveling stations. The signal s_p at any grid point p can be calculated by

$$s_p = \boldsymbol{c}_p^T(\boldsymbol{C} + \boldsymbol{D})^{-1} d\boldsymbol{N} \tag{4}$$

where $\boldsymbol{c}_p$ is a vector whose elements are the covariance of the signal at grid point p and those at all GPS/leveling stations, $\boldsymbol{C}$ and $\boldsymbol{D}$ are the covariance matrix of signals among the stations and the variance matrix of noises at the stations, respectively. The difficulty with the LSC lies in the selection of a covariance function. The empirical method can work, but needs large amount of GPS/leveling data to get a meaningful and reliable covariance function. Alternatively, one may arbitrarily select a function, e.g., Gauss function, as covariance function, but the results will vary with the selection. Fukuda *et al.* (1997) and Featherstone (2000), for instance, employed the LSC approach to refine the geoid of Japan JGEOID93 and the geoid of Perth region in Australia, respectively.

The second method is to use a multi-quadratic interpolation function (Hardy, 1975) to establish a refined geoid model. The function reads

$$s(x, y) = \sum_{j=1}^{t} \alpha_j \theta(x, y; x_j, y_j) \tag{5}$$

where $s(x, y)$ is the signal at point (x, y), $\theta(x, y; x_j, y_j)$ is a kernel function (or a surface), t is number of data points used in the interpolation, and α_j the coefficients to be determined from data points. The surfaces mentioned in Hardy (1975) include circular hyperboloids in two sheets, circular paraboloids, and right cones.

The third method is the weighted average. Corrections (or signals) to the transformed gravimetric geoid heights at grid points can be computed using dN at the GPS/leveling stations with a weighted average method. There are various weighting schemes.

It should be emphasized that there are several other methods available, like the finite element method (FEM), the Fourier series, the higher order polynomial fitting, the continuous curvature splines in tension, etc. (e.g., Featherstone, 2000; Fotopoulos *et al.*, 2003), but no one can be claimed spurious over others for all cases. It much depends on the feature of a local geoid model, and the distribution and accuracy of the observed geoid heights. The selection of a method to be used must be done through empirical tests. As an example, we conducted tests using Hong Kong data to

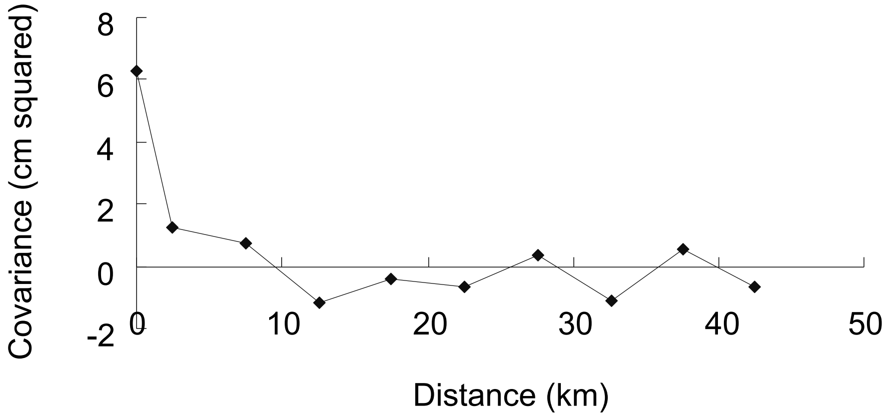

Fig. 1. The empirical covariance function.

Table 1. Estimated geoid heights at checkpoints with the three methods (in meter).

Checkpoint	Observed geoid height	The weighted average	The LSC	The multi-quadric function
141	−1.179	−1.164	−1.176	−1.177
142	−0.996	−1.021	−1.031	−1.032
139	−1.296	−1.300	−1.301	−1.317
72	−1.027	−1.039	−1.041	−1.051
94	−2.224	−2.220	−2.211	−2.215
240	−2.286	−2.265	−2.264	−2.259
Max. difference		0.025	0.035	0.036
Min. difference		−0.021	−0.022	−0.027
RMS of differences		0.016	0.019	0.023

see which of the above three methods can provide better results for the area. 31 GPS/leveling stations whose observed geoid heights have better accuracy were selected and 6 of them were used as checking points. The above-mentioned three methods were evaluated with the following details:

1) The weighted average method: the weight is defined as $P_i = (1/r_i)^3$, where r_i is the distance between the computation point and running point. The power of 3 was selected from empirical tests;

2) The least square collocation: the discrepancies dN at the 31 GPS stations were used to formulate an empirical covariance function. Figure 1 is a plot of the covariance function approximated with the cubic-spline;

3) The circular hyperboloid with $\delta = 5$ km was used as surface in the multi-quadric method, i.e., $\theta(x, y; x_j, y_j) = [(x - x_j)^2 + (y - y_j)^2 + \delta^2]^{1/2}$.

The results are given in Table 1, where the difference is the observed value minus the estimated. It can be seen from the table that the weighted average method provides a better result and at the same time is simpler.

3. Case Study

The authors completed the project of the determination of Hong Kong local geoid HKGEOID-2000 funded by the Hong Kong Research Grant Council and worked as consultants for the project of the determination of Shenzhen geoid SZGEOID-2001, funded by the Department of Lands, Government of Shenzhen, China. The developed methodology and software for the Hong Kong project were used in the Shenzhen project.

3.1 Data used for the HKGEOID-2000 and SZGEOID-2001

Hong Kong is a hilly territory with area about 1000 km^2. Its DTM with resolution of 100 m was created from 1:20000 topographic maps. The DTM of its neighboring region Shenzhen was also obtained. 55 GPS/leveling stations evenly distributed in the territory were used. Their ellipsoidal heights are referenced to WGS84 ellipsoid, and their leveled heights above the Hong Kong Principal Datum. According to the data provider Surveying and Mapping Office of the Hong Kong Government, the estimated accuracy of so derived geoid height ranges from 1–4 cm (less accurate for some points on the tops of hills whose heights were determined with precise trigonometric leveling). The gravity data include 640 gravity observations in Hong Kong territory and 2158 measurements in its neighboring region Shenzhen. The gravity measurements in Hong Kong are of station spacing 2 km on land and 2–4 km in sea. They were collected using Lacoste and Romberg model ‘G’ land gravity meter and model ‘H/U’ seabed gravity meter. The gravity measurements in Shenzhen were collected in 2001 with Lacoste and Romberg model ‘G’ and ‘D’ land gravimeter and model ‘S’ sea gravimeter. Figure 2 shows the distribution of these grav-

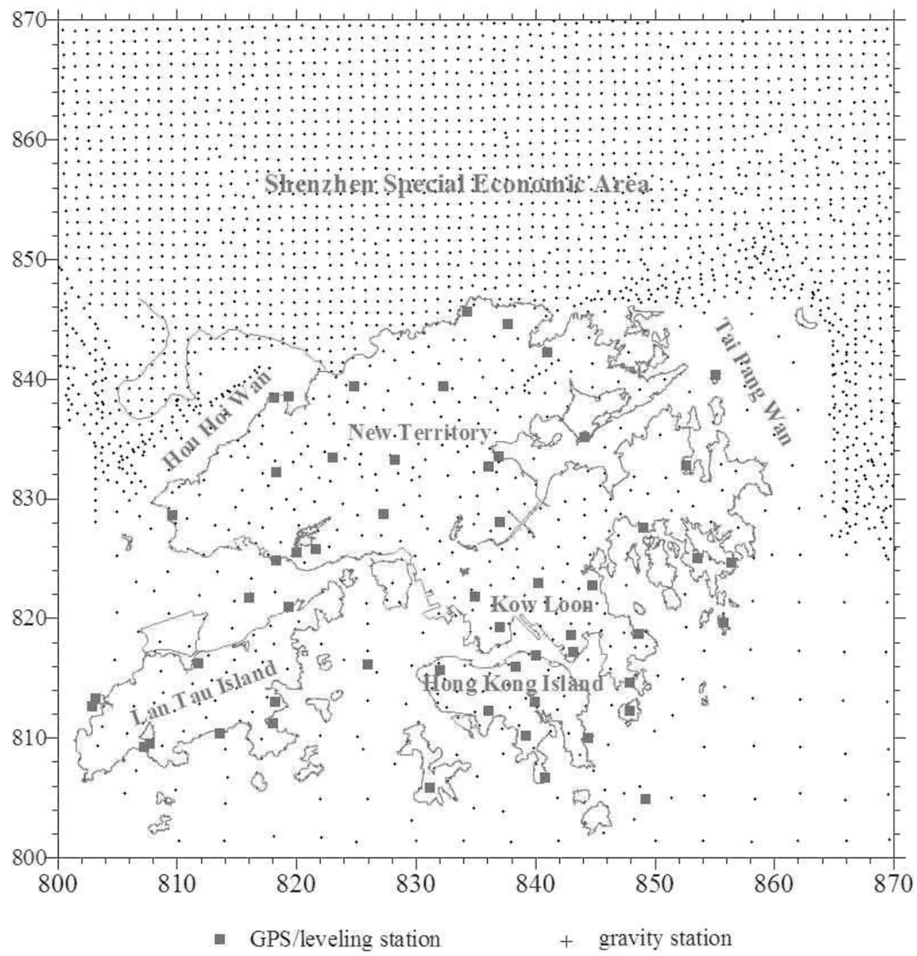

Fig. 2. Distribution of GPS/leveling stations and gravity measurement points (the coordinates in km are in Hong Kong 1980 grid system).

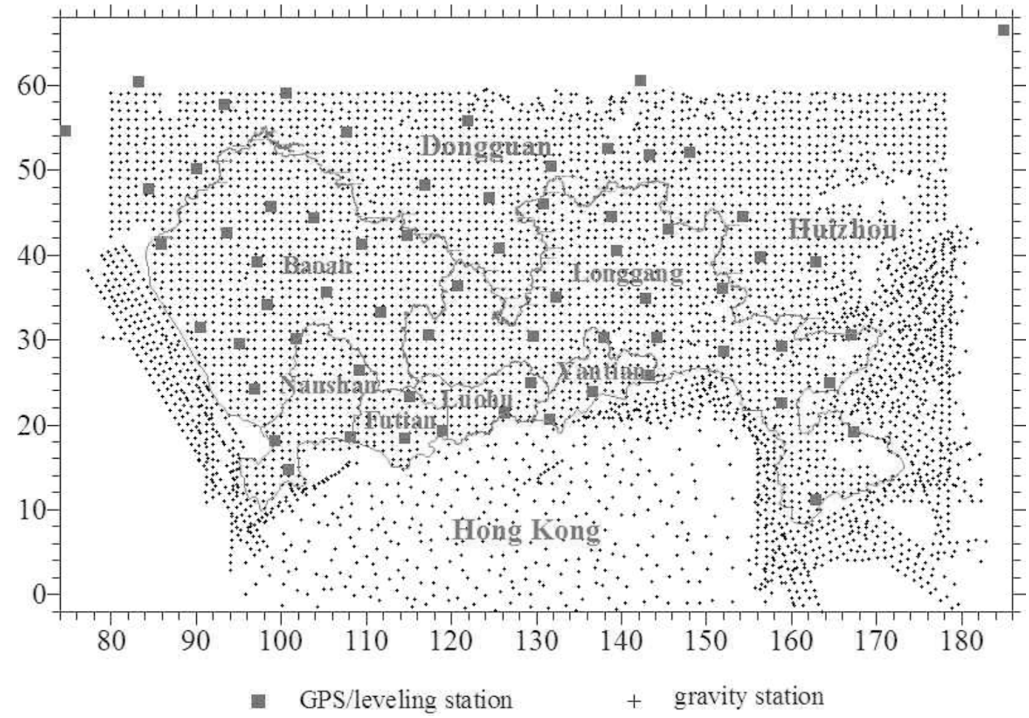

Fig. 3. Distribution of GPS/leveling Stations and gravity measurement points (the coordinates in km are in Shenzhen grid system).

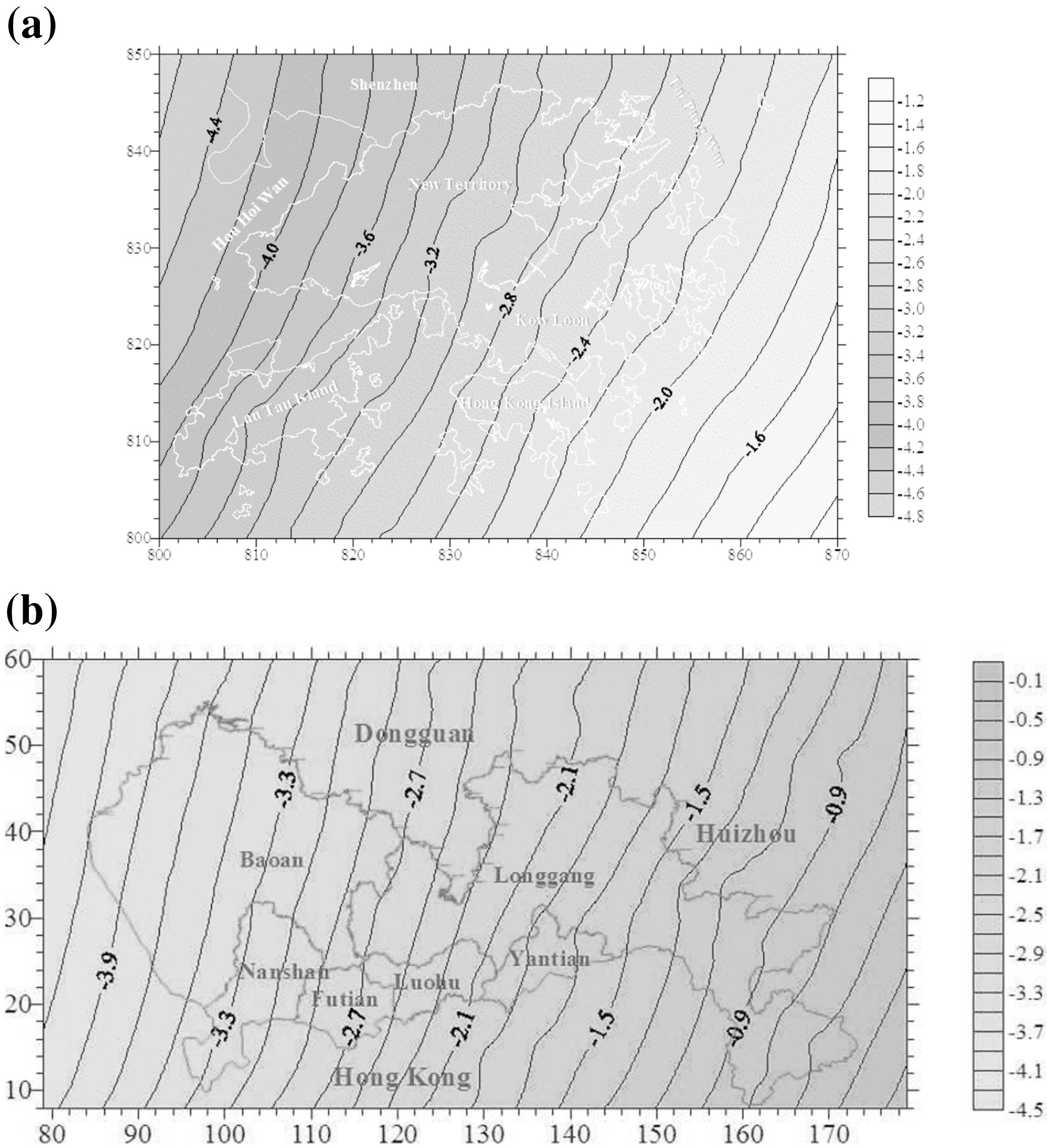

Fig. 4. (a) HKGEOID-2000. (b) SZGEOID-2001.

ity measurements and GPS/leveling stations.

Shenzhen is one of the special economic zones in China with area of 2020 km^2. It is on the north of Hong Kong. There are 65 GPS/Leveling stations with average spacing of 10 km. According to the data provider, Department of Lands of Shenzhen Government, the observed geoid heights are of accuracy of 1–2 cm. The GPS-determined heights are referred to WGS-84 ellipsoid and the leveled heights are in normal height system and referred to the 1956 Yellow Sea Datum. The gravity measurements used include 3608 observations on land with accuracy of 0.1 mGal and 1262 in sea with accuracy of 1.6 mGal. The gravity stations are spaced by 1 km. In the determination of Shenzhen geoid 298 gravity measurements on land and 45 in sea in Hong Kong territories were included. Figure 3 shows the distribution of these gravity measurement points and GPS/leveling stations.

3.2 Construction of HKGEOID-2000 and SZGEOID-2001

Both Hong Kong and Shenzhen geoid models were constructed in the same manner as discussed in Section 2, i.e., the three-step procedure. The remove-restore technique was used to construct their gravimetric geoid models. In this step the study was made to select a better geo-potential model for the region (Luo and Chen, 2002). Three models EGM96, WDM94 (Ning *et al.*, 1994) and GPM98CR were tested using the GPS/leveling data and gravity measurements in the region. The results suggested that the WDM model was the best among the three and therefore was used in these two projects. This may be because the gravity measurements in China were used in the development of WDM94. In the second step the 3-parameter transformation of the gravimetric geoid was performed using the least squares fitting. The

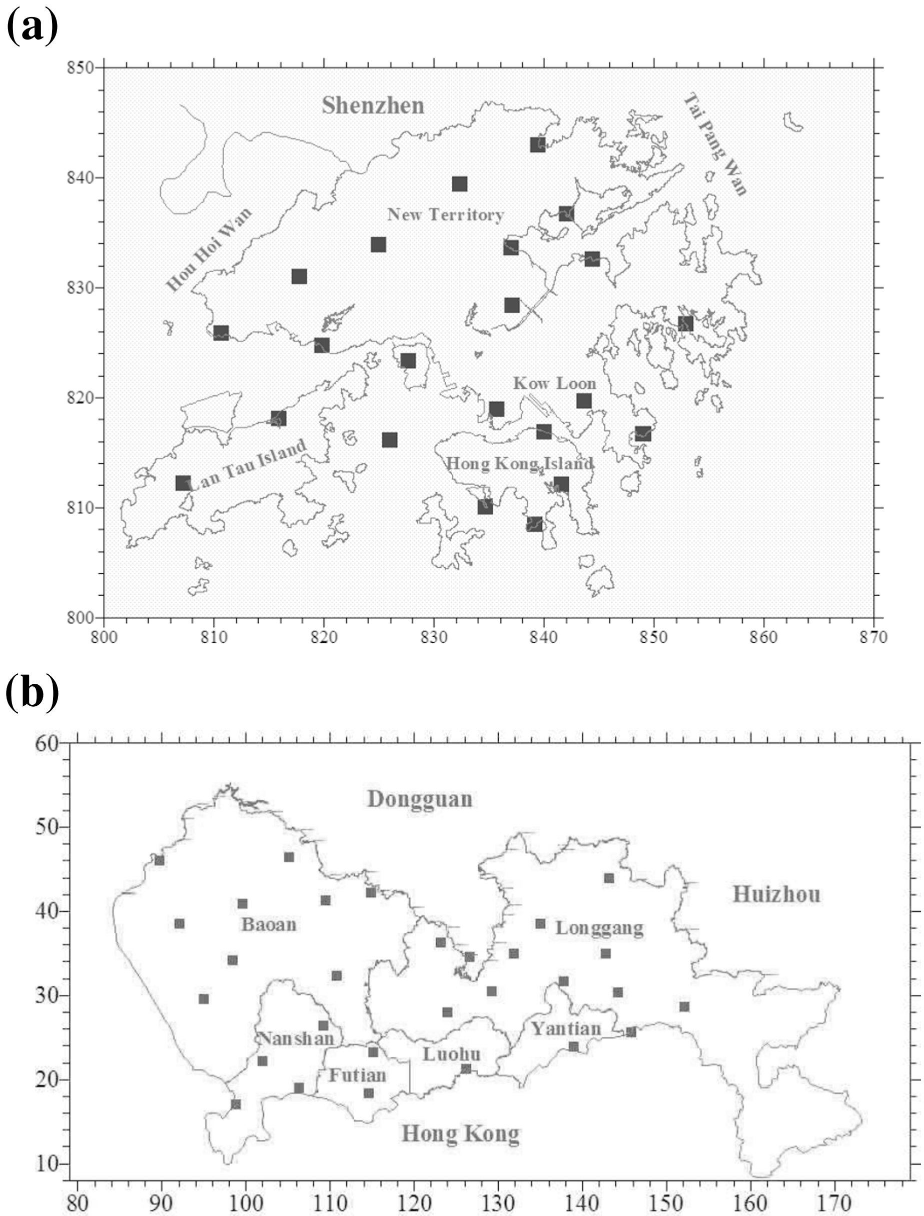

Fig. 5. (a) Distribution of Check Points in Hong Kong (Hong Kong coordinate system in km). (b) Distribution of Check Points in Shenzhen (Shenzhen coordinate system in km).

transformed gravimetric geoid height N' and the original gravimetric geoid height N are related as

$$N'(x, y) = N(x, y) + a_0 + a_1 x + a_2 y \tag{6}$$

where x and y are grid coordinates of a point, a_0, a_1, and a_2 are bias, tilt in the x-direction, and tilt in the y-direction, respectively. The transformed geoid model is finally refined using a special weighted average, called local Shepard surface fitting method:

$$s(x_P, y_P) = \begin{cases} \sum_{i=1}^{n} dN(x_i, y_i) \cdot K(r_i)^m / \sum_{i=1}^{n} K(r_i)^m, & r_i \neq 0 \\ dN(x_i, y_i), & r_i = 0 \end{cases} \tag{7}$$

where dN is the difference between the observed geoid height and the transformed gravimetric geoid height, r_i is the distance between interpolating point (x_P, y_P) (grid points) and known point (x_i, y_i) (GPS/leveling stations), m the fitting power should be integer ($m = 1, 2, \cdots$), and $K(r_i)$ the kernel function or so-called weighting function, i.e.,

$$K(r) = \begin{cases} 1/r, & 0 < r \leq D/3 \\ \frac{27}{4D}(\frac{r}{D} - 1)^2, & D/3 < r \leq D \\ 0, & r > D \end{cases} \tag{8}$$

with D being interpolating or searching radius. The value s at a grid point is then added to the transformed geoid height, resulting in the refined geoid model. The values D and m should be selected through testing. There are two reasons for employing the local Shepard surface fitting method for these two study areas. One is based on the tests with Hong Kong data as discussed in Section 2.2, which suggests the weighted average provided a better result. The local Shepard surface fitting is a kind of weighted average method. The difference only lies in kernel (or weighting) function $-K(r)$

for the Shepard fitting and $1/r$ for the simple weighting scheme used in Section 2.2. The other reason is that we consider that the signals are of local nature and therefore the dN at points closer to interpolating point should have much more significant contributions. One can see from Eq. (8) that when $r > D/3$ the weight function $K(r)$ is smaller than that of the simple weighting scheme, which follows more closely the above consideration. With the Shenzhen data we have found that the better results can be achieved if $m = 3$ and $D = 50$ km. These values were used for both study areas. Actually, there should be no much difference between the simple weighting scheme and the Shepard fitting method for the study areas, for the selected m of 3 is larger. The simple weighting scheme was also applied to the Shenzhen data and did not produce significantly different results. The final geoid models for both regions are given in Fig. 4(a) and (b).

3.3 Evaluation and analysis of the accuracy of the two geoid models

The constructed geoid models HKGEOID-2000 and SZGEOID-2001 were thoroughly evaluated before they were released for practical uses. The authorities concerned did independent and accurate GPS and leveling surveys at 22 stations in Hong Kong and 29 in Shenzhen, which do not coincide with the stations used for the construction of the geoid models. They are referred as checkpoints. Figure 5(a) and (b) show their distribution.

A geoid model was evaluated with two measures: absolute agreement and relative agreement with the observed geoid heights. These two agreements were evaluated for both the transformed gravimetric geoid and refined geoid models (the final models), from which the accuracy of the final geoid models and the efficiency of the signal extraction process can be obtained. Table 2 lists the statistics of the residuals after the least squares fitting of the gravimetric geoid models into observed geoid heights. The residuals are the discrepancies between the observed geoid heights and the transformed gravimetric geoid model. The standard deviation measures the absolute agreement of the transformed gravimetric geoid model with the GPS/leveling data. The absolute agreement was also evaluated at checkpoints. The differences between the observed geoid heights and the transformed gravimetric geoid heights at checkpoints were computed and their statistics is listed in Table 3. The standard deviation also measures the absolute agreement of the transformed gravimetric geoid model with GPS/leveling data. The value in Table 3 can be considered as external measure, because the observed geoid heights at checkpoints are not included in the construction of a geoid model; while the value in Table 2 is regarded as internal measure. To assess the efficiency of the proposed hybrid approach, the differences between the observed geoid heights and the refined geoid heights at the checkpoints were computed. Table 4 gives their statistics in the sense of absolute agreement. In order to evaluate the relative agreement of a geoid model with GPS/leveling data, the geoid height differences among the checkpoints were computed for both the transformed gravimetric geoid models and final geoid models and then compared with the observed ones. The results are given in Tables 5 and 6. From the results following observations can be made:

Table 2. Statistics of the residuals after the 3-parametr least squares fitting (unit: m).

	Max	Min	STD
Hong Kong	0.056	−0.081	0.026
Shenzhen	0.051	−0.083	0.028

Table 3. Statistics of the differences between the transformed gravimetric geoid heights and the observed ones at checkpoints (unit: m).

	# of check points	Max	Min	STD
Hong Kong	22	0.034	−0.053	0.022
Shenzhen	29	0.041	−0.061	0.019

Table 4. Statistics of the differences between the final geoid heights and observed ones at check points (unit: m).

	# of check points	Max	Min	STD
Hong Kong	22	0.031	−0.025	0.017
Shenzhen	29	0.026	−0.022	0.014

1) The external measure of the absolute agreement of the transformed geoid with GPS/leveling data is very similar for both areas, i.e., 22 mm for Hong Kong and 20 mm for Shenzhen. The internal measure is also similar for both areas, but a little bit larger than the external one, i.e., 27 mm for Hong Kong and 28 mm for Shenzhen.

2) The absolute agreement of the final geoid model HKGEOID-2000 with the GPS/leveling data at the checkpoints is 17 mm, and 14 mm for SZGEOID-2001. The refinement process (step 3) improved the agreement from 22 mm to 17 mm by 23% for Hong Kong geoid, and from 20 mm to 14 mm by 30% for Shenzhen geoid.

3) Improvement of the relative agreement is also significant. The overall relative agreement for Hong Kong geoid is improved from 2 ppm to 1.7 ppm by 15%, and for Shenzhen from 1.4 ppm to 1.1 ppm by 21%.

4) The above (2) and (3) indicate that the proposed signal extraction method works well and can significantly improve the accuracy of geoid.

5) The above values of agreement are affected by the errors of observed geoid heights at checking points. To estimate the accuracy of the constructed geoid models we must know the errors of the observed geoid heights. For simplicity we assume that the observed geoid heights at checkpoints have similar size of error as the constructed geoid model. With such an assumption, the HKGEOID-2000 has absolute accuracy of 12 mm and overall relative accuracy of 1.2 ppm; the SZGEOID-2001 has absolute accuracy of 10 mm and relative accuracy of 0.8 ppm.

6) The SZGEOID-2001 has higher accuracy than HKGEOID, though both achieve centimeter accuracy. This was expected, for Shenzhen Government invested a

Table 5. Statistics of the differences between modeled and observed geoid height differences among 22 checkpoints in Hong Kong.

Baseline Length (km)	No. of height diff.	RMS-1 (mm)	Relative-1 (ppm)	RMS (mm)	Relative (ppm)
0∼5	5	18	7.2	19	7.6
5∼10	27	27	3.6	21	2.8
10∼15	21	26	2.1	25	2.0
15∼20	33	32	1.8	21	1.2
20∼25	28	40	1.8	29	1.3
25∼30	33	35	1.3	26	0.9
30∼35	22	26	0.8	25	0.8
Overall Statistics	169	31*	2.0*	24*	1.7*

Note that: (1) columns RMS-1 and relative-1 are referred to the transformed gravimetric geoid; while columns RMS and relative are referred to the final geoid; (2) * weighted average value with weight being the number of height differences.

Table 6. Statistics of differences between modeled and observed geoid height differences among 29 checkpoints in Shenzhen.

Baseline Length (km)	No. of height diff.	RMS-1 (mm)	Relative-1 (ppm)	RMS (mm)	Relative (ppm)
0∼5	5	21	8.4	17	6.8
5∼10	47	24	3.2	19	2.5
10∼15	54	21	1.7	19	1.5
15∼20	60	22	1.3	19	1.1
20∼25	55	24	1.1	18	0.8
25∼30	49	27	1.0	18	0.7
30∼35	43	30	0.9	19	0.6
35∼40	31	31	0.8	20	0.5
40∼45	25	38	0.9	17	0.4
45∼50	16	30	0.6	15	0.3
>50	21	48	0.9	26	0.5
Overall Statistics	406	27*	1.4*	19*	1.1*

Note that: (1) columns RMS-1 and relative-1 are referred to the transformed gravimetric geoid; while columns RMS and relative are referred to the final geoid; (2) * weighted average value with weight being the number of height differences.

great deal to resurvey GPS and leveling and conduct dense gravity measurements for the purpose, while Hong Kong geoid was constructed only through collecting existing data which were used for other purposes.

4. Conclusion Remarks

Advantage of all data/information must be taken as much as possible to construct a precise geoid model. The proposed methodology of three-step sequential processing procedure is simple and works well. It involves the construction of a gravimetric geoid using the well-known remove-restore technique, the least squares fitting of the gravimetric geoid with respect to GPS/leveling data to remove bias and tilts (3-parameter transformation), and the signal extraction to further improve the transformed gravimetric geoid. The last step is of particular importance to construct a precise geoid model. There are various methods for the purpose of signal extraction, but no one can be claimed superior over others for all the cases. Careful tests on an area of interest must be conducted to select a better method. The study of two areas, Hong Kong and Shenzhen, shows that refinement process (the signal extraction) can improve the absolute agreement of Hong Kong and Shenzhen geoid models with their GPS/leveling data by 23% and 30%, respectively; while it improves the relative agreement of Hong Kong and Shenzhen geoid models by 15% and 21%, respectively. This improvement is significant. Using the proposed procedure, both HKGEOID-2000 and SZGEOID-2001 achieve the level of centimeter accuracy and about 1 ppm relative accuracy, which is good for GPS leveling.

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References

Doerflinger, E., Z. Jiang, H. Duquenne, and R. bayer, Determination of the quasi-geoid in a mountainous area: example of eastern Pyrenees (France), in *Gravity, Geoid and Marine Geodesy,* edited by J. Segawa, H. Fujimoto, and S. Okubo, pp. 643–650, Springer, 1997.

Engelis, T., R. H. Rapp, and Y. Bock, Measuring orthometric height differences with GPS and gravity data, *Manuscripta Geodaetica,* **10**, 187–194, 1985.

Featherstone, W. E., Refinement of gravimetric geoid using GPS and leveling data, *Journal of Surveying Enginnering,* **126**(2), 27–58, 2000.

Featherstone, W. E., M. C. Dentith, and J. F. Kirby, Strategies for the accurate determination of orthometric heights from GPS, *Survey Review,* **34**(267), 278–296, 1998.

Forsberg, R., J. Kaminskis, and D. Solheim, Geoid of the Nordic and Baltic region from gravimetry and satellite altimetry, in *Gravity, Geoid, and Marine Geodesy,* edited by J. Segawa, H. Fujimoto, and S. Okubo, pp. 540–547, Springer, 1997.

Fotopoulos, G., C. Kotsakis, and M. G. Sideris, A new Canadian geoid model in support of leveling by GPS, *Geomatica,* **53**(4), 53–62, 1999.

Fotopoulos, G., W. E. Featherstone, and M. G. Sideris, Fitting a gravimetric geoid model to the Australia height datum via GPS data, Manuscript obtained from the second author, 2003.

Fukuda, Y., J. Kuroda, Y. Takabatake, J. Itoh, and M. Murakami, Improvement of JGEOID 93 by the geoidal heights derived from GPS/leveling surveys, in *Gravity, Geoid, and Marine Geodesy,* edited by J. Segawa, H. Fujimoto, and S. Okubo, pp. 589–596, Springer, 1997.

Hardy, R. L., Research results in the application of multi-quadric equations to surveying and mapping problems, *Surveying and Mapping,* **35**(12), 321–332, 1975.

Luo, Z. C. and Y. Q. Chen, Evaluation of geo-potential models EGM96, WDM94, and GPM98CR in Hong Kong and Shenzhen region, *Journal of Geo-spatial Engineering,* **4**(1), 21–30, 2002.

Moritz, H., Local geoid determination in mountainous areas, OSU Rep. No 353, Department of Geodetic Science and Surveying, the Ohio State University, Ohio, USA, 1983.

Ning, J. S., J. C. Li, D. B. Chao, and Z. L. Guan, The Research of the earth's gravity field model WDM94, *Journal of Wuhan Technical University of Surveying and Mapping,* **19**(4), 283–291, 1994.

Torge, W., *Geodesy*, third edition, Walter de Gruyter, 416 pp, 2001.

Yanalak, M. and O. Baykal, Transformation of ellipsoid heights to local leveling heights, *Journal of Surveying Engineering,* **127**(3), 90–103, 2001.

Yang, Z. J. and Y. Q. Chen, Determination of local geoid with the geometric method—a case study, *Journal of Surveying Engineering,* **125**(3), 136–146, 1999.

Yang, Z. J. and Y. Q. Chen, Determination of Hong Kong gravimetric geoid, *Survey Review,* **36**(279), 23–34, 2001.

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