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Formulation and analytic calculation for the spin angular momentum of a moonlet due to inelastic collisions of ring particles

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Small moonlets embedded in planetary rings acquire spin angular momentum by inelastic collisions of a number of ring particles. We obtain analytic expressions for the mean and mean square spin angular momenta delivered to a moonlet in the high-velocity case where mutual gravity between the moonlet and particles can be neglected. We find that the mean angular momentum brought by a large number of small impacts would result in an equilibrium rotation of a moonlet in the prograde direction that is slower than the synchronous rotation, while large impacts would significantly affect the rotation when the mass of largest impactors is comparable to the moonlet's mass and/or the velocity dispersion of particles is larger than the Kepler shear across the moonlet's radius. We present a new formulation that allows a unified analysis of these two components of moonlet rotation, and confirmed the validity of this formulation and the above analytic calculations using N-body simulation.

Key words: Celestial mechanics, rotational dynamics, planetary rings, origin of solar system.

1. Introduction

A number of small satellites have been found in Saturn's rings, and it is believed that there are a large number of unseen moonlets embedded in the rings. Isolated large moons ($\gtrsim$ 100 m) would be tidally despun to rotate synchronously, but smaller moonlets embedded in rings must be spun up by collisions with other particles (Weidenschilling *et al.*, 1984). When particles' velocity dispersion is much larger than the escape velocity of a moonlet, mutual gravity between particles and the moonlet can be neglected, and we can evaluate analytically the angular momentum delivered to the moonlet by collisions of particles. In previous studies on planetary rotation, mean and mean square angular momenta that a planet acquires by accretion of planetesimals have been evaluated both analytically and numerically (e.g., Giuli, 1968; Tanikawa *et al.*, 1989; Ida and Nakazawa, 1990; Lissauer and Kary, 1991; Lissauer and Safronov, 1991; Dones and Tremaine, 1993a, b; Lissauer *et al.*, 1997; Ohtsuki and Ida, 1998; see a review by Lissauer *et al.*, 2000). In these studies, it is assumed that planetesimals are accreted by the planet whenever they collide. However, in ring-satellite systems where the tidal force of the central planet is important, inelastic rebound of particles needs to be taken into account.

On the other hand, it has been shown that two components need to be taken into account in the study of planetary rotation: One is the systematic component of rotation that a planet obtains from a disk of small planetesimals, and the other is the random component imparted by large impactors. Dones and Tremaine (1993a, b) derived a formulation for planetary rotation due to planetesimal accumulation that allows a unified treatment of the two components. Previous studies related to rotation of ring particles often assume that they have identical sizes (e.g., Araki, 1988, 1991). Salo (1987a, b) studied rotation of ring particles using analytic calculation and numerical simulation. In the analytic calculation, Kepler motion of particles was taken into account in an approximate manner, by assuming an isotropic distribution of random velocities. He primarily studied the case of identical particles: the case of a power-law size distribution was also examined, but the relative contribution of the systematic and the random components was not discussed. Recently, Morishima and Salo (2004) obtained the systematic component of moonlet rotation by three-body orbital integration, but they did not evaluate the random component. In order to obtain the two components for moonlet rotation, we need to modify the previous formulation of Dones and Tremaine (1993a, b), which was derived for planetary rotation under the assumption of perfect accretion.

In the present paper, we carry out analytic calculation for the spin angular momentum delivered to a moonlet by collisions of a number of particles, neglecting their mutual gravity. Section 2 describes basic formulation for the analytic calculation, and we present the calculation for the mean and mean square angular momenta in Section 3. In Section 4, we derive a new formulation that allows a unified analysis of the systematic and the random components of moonlet rotation due to inelastic rebound of particles, and we use the results of Section 3 to examine the relative importance of the two components. We find that the contribution of the random component is significant when the mass of largest impactors is comparable to the moonlet's mass and/or the velocity dispersion of particles is larger than the Kepler shear across the moonlet's radius. We confirm the validity of the new formulation and these analytic results using N-body simulation in Section 5. Our results are summarized in Section 6.

2. Basic Formulation for the Calculation of Spin Angular Momentum

We assume that a moonlet (subscript 1) and a particle (subscript 2) are spheres with the same density, and that their masses, radii, and spin angular velocity vectors are given by m_i, R_i, and $\boldsymbol{\omega}_i$ $(i = 1, 2)$, respectively. We also assume that the moonlet is on a circular orbit with a semimajor axis a and the Keplerian angular velocity Ω. We use Hill's coordinate system centered on the moonlet (e.g., Nakazawa *et al.*, 1989), where the x-axis points radially outward, the y-axis points in the direction of the moonlet's orbital motion, and the z-axis is normal to the x-y plane. The masses of the moonlet and a particle are assumed to be much smaller than the mass of the central planet M_c, and eccentricities and inclinations of particles are assumed to be much smaller than unity. Then, equations for the motion of a particle relative to the moonlet can be linearized, and they can be written in nondimensional forms if time is scaled by Ω^{-1} and length by the Hill radius $R_H = ah$, where $h \equiv \{(m_1 + m_2)/3M_c\}^{1/3}$. Furthermore, the motions of the moonlet and a particle can be separated into the relative motion and the center of mass motion, and the nondimensional equations for the relative motion are written as

$$\begin{aligned} \ddot{\tilde{x}} &= 2\dot{\tilde{y}} + 3\tilde{x} - 3\tilde{x}/\tilde{r}^3, \\ \ddot{\tilde{y}} &= -2\dot{\tilde{x}} - 3\tilde{y}/\tilde{r}^3, \\ \ddot{\tilde{z}} &= -\tilde{z} - 3\tilde{z}/\tilde{r}^3, \end{aligned} \tag{1}$$

where tildes are used to denote scaled quantities, and $\tilde{r} \equiv (\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2)^{1/2}$.

In this paper, we consider the case where mutual gravity between the moonlet and particles can be neglected. Ohtsuki (1992, 1999) demonstrated that analytic expressions of collision and velocity stirring rates derived for non-gravitating ring particles agree well with results of three-body orbital integration that takes into account mutual gravity when particles' random velocity is larger than a few times their escape velocity. Such a high velocity would be realized in rings perturbed by other larger satellites. We also confirmed that the results on moonlet rotation presented in this paper agree with three-body orbital integration in such a high-velocity case (Ohtsuki, 2004), although the effect of mutual gravity enhances the rate of angular momentum transfer during particle collisions in lower velocity cases (Morishima and Salo, 2004; Ohtsuki, 2004). However, analytic studies for the above high-velocity limit are useful to understand the numerical results of orbital integration for the gravitating case. They would be also applied to asteroid rotation, where the effect of mutual gravity is less important.

In the non-gravitating case, the last terms in the r.h.s. of Eq. (1) (i.e., those proportional to $-3/\tilde{r}^3$) can be omitted, and the relative motion of particles can be written as

$$\begin{aligned} \tilde{x} &= \tilde{b} - \tilde{e}\cos(\tilde{t} - \tau), \\ \tilde{y} &= -\frac{3}{2}\tilde{b}(\tilde{t} - \phi) + 2\tilde{e}\sin(\tilde{t} - \tau), \\ \tilde{z} &= \tilde{i}\sin(\tilde{t} - \varpi), \end{aligned} \tag{2}$$

where $\tilde{e}$ and $\tilde{i}$ are the eccentricity and the inclination for the relative motion scaled by h; $\tilde{b}$ is the difference in the semimajor axes of the moonlet and the particle scaled by R_H; ϕ defines the origin of time; and τ and ϖ are the horizontal and vertical phase angles, respectively.

First, we describe the formulation and the analytic results for the collision rate of ring particles onto a moonlet based on previous works on planetary accretion rates (Nakazawa *et al.*, 1989; Greenzweig and Lissauer, 1990, 1992), which we will use below. The nondimensional collision rate of particles onto a moonlet can be defined as (Nakazawa *et al.*, 1989)

$$P_{col}(\tilde{e}, \tilde{i}) \equiv \int p_{col}(\tilde{e}, \tilde{i}, \tilde{b}, \tau, \varpi)\frac{3}{2}|\tilde{b}|d\tilde{b}\frac{d\tau d\varpi}{(2\pi)^2}, \tag{3}$$

where p_{col} is the collision probability between the moonlet and particles with given $\tilde{e}$, $\tilde{i}$, and $\tilde{b}$, and $p_{col} = 1$ for collision orbits and 0 otherwise. In the high-velocity case where mutual gravity can be neglected, p_{col} can be evaluated analytically as (Nakazawa *et al.*, 1989)

$$\begin{aligned} p_{col}(\tilde{e}, \tilde{i}, \tilde{b}) &= \int p_{col}(\tilde{e}, \tilde{i}, \tilde{b}, \tau, \varpi)\frac{d\tau d\varpi}{(2\pi)^2} \\ &= \frac{2\tilde{r}_p^2}{3\pi\tilde{b}\tilde{i}}\sqrt{\frac{\tilde{e}^2 + \tilde{i}^2 - \frac{3}{4}\tilde{b}^2}{\tilde{e}^2 - \tilde{b}^2}}, \end{aligned} \tag{4}$$

where $\tilde{r}_p \equiv (R_1 + R_2)/R_H$. Then, performing the integral over $\tilde{b}$, P_{col} can be obtained as (Nakazawa *et al.*, 1989; Greenzweig and Lissauer, 1990)

$$\begin{aligned} P_{col} &= \int p_{col}(\tilde{e}, \tilde{i}, \tilde{b})\frac{3}{2}|\tilde{b}|d\tilde{b} \\ &= \frac{2\tilde{r}_p^2 E(k)}{\pi}\sqrt{1 + (\tilde{e}/\tilde{i})^2}, \end{aligned} \tag{5}$$

where $E(k)$ is the complete elliptic integral of the second kind with $k = \left[3\tilde{e}^2/4(\tilde{e}^2 + \tilde{i}^2)\right]^{1/2}$.

On the other hand, N-body simulations show that eccentricities and inclinations of particles in optically thin rings follow the Rayleigh distribution (e.g., Ohtsuki and Emori, 2000) given as

$$f(\tilde{e}, \tilde{i})d\tilde{e}d\tilde{i} = \frac{4\tilde{e}\tilde{i}}{\langle\tilde{e}^2\rangle\langle\tilde{i}^2\rangle}\exp\left(-\frac{\tilde{e}^2}{\langle\tilde{e}^2\rangle} - \frac{\tilde{i}^2}{\langle\tilde{i}^2\rangle}\right)d\tilde{e}d\tilde{i}. \tag{6}$$

The Rayleigh distribution average of P_{col} can be calculated as

$$\langle P_{col}\rangle = \int P_{col}(\tilde{e}, \tilde{i})f(\tilde{e}, \tilde{i})d\tilde{e}d\tilde{i}. \tag{7}$$

In the high-velocity case, $\langle P_{col}\rangle$ can be evaluated analytically (Greenzweig and Lissauer, 1992; Dones and Tremaine, 1993b) as

$$\langle P_{col}\rangle = \frac{\tilde{r}_p^2}{\pi}I(\beta), \tag{8}$$

where $\beta \equiv \langle\tilde{i}^2\rangle^{1/2}/\langle\tilde{e}^2\rangle^{1/2}$ and

$$I(\beta) \equiv \int_0^1 \frac{4E\left(\frac{\sqrt{3}}{2}\sqrt{1 - \chi^2}\right)}{\left\{\beta + (\beta^{-1} - \beta)\chi^2\right\}^2}d\chi. \tag{9}$$

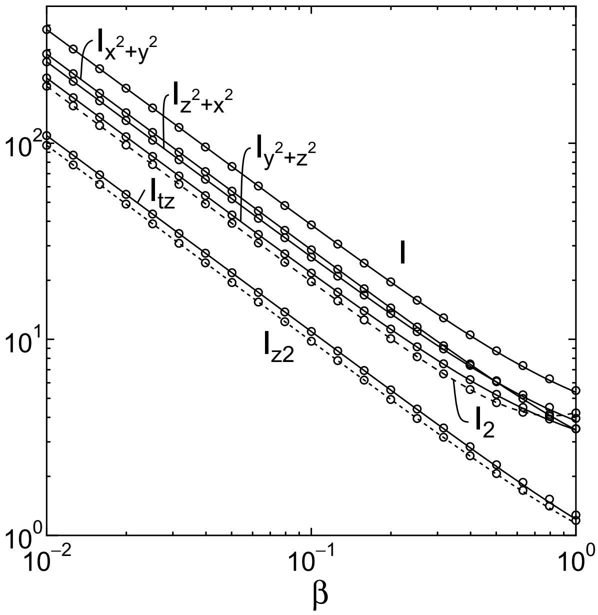

Fig. 1. The plots of the integrals appearing in the analytic expressions. The solid lines (I, I_{tz}, $I_{x^2+y^2}$, $I_{y^2+z^2}$, and $I_{z^2+x^2}$) and the dashed lines (I_2 and I_{z2}) represent the results obtained by direct numerical integration of these integrals, while the open circles show the results calculated by the approximate analytic expressions described in Appendix (Eqs. (A.7), (A.9), and (A.13)).

The plot of $I(\beta)$ is shown in Fig. 1. An approximate analytic expression for $I(\beta)$ is derived by Greenzweig and Lissauer (1992), and is also plotted by the open circles. The method of derivation of the analytic expression is described in Appendix.

The rate of accumulation of spin angular momentum can be defined in a similar manner. Suppose that a moonlet and a particle collide with each other with the relative velocity of the centers of the two bodies in the rotating coordinate system $\boldsymbol{v}$, and that the position of the particle relative to the moonlet at impact is $\boldsymbol{r}$. In general, collisions change the normal and the tangential components of $\boldsymbol{v}$ to the tangent plane. However, as we show below, the change of spin angular momentum of a non-gravitating moonlet depends only on the change of the tangential component of the relative velocity (in the rotating coordinate system), $\boldsymbol{v}_\mathrm{t}$. The tangential component of the relative velocity of the two contacting points at impact can be written as (e.g., Araki and Tremaine, 1986)

$$\boldsymbol{u}_\mathrm{t} = \boldsymbol{v}_\mathrm{t} + \boldsymbol{\Omega} \times \boldsymbol{r} + \boldsymbol{\lambda} \times (R_1\boldsymbol{\omega}_1 + R_2\boldsymbol{\omega}_2), \tag{10}$$

where $\boldsymbol{\lambda} \equiv \boldsymbol{r}/|\boldsymbol{r}|$ is the unit vector pointing from the center of the moonlet to that of the particle, and $\boldsymbol{\Omega} = (0, 0, \Omega)$ is the Keplerian angular velocity vector of the moonlet. In the above, we used the relation $\boldsymbol{r} = (R_1 + R_2)\boldsymbol{\lambda}$ at impact. We introduce the tangential restitution coefficient ε_t, where $-1 \le \varepsilon_\mathrm{t} \le 1$ (perfectly smooth spheres have $\varepsilon_\mathrm{t} = 1$). Then, the tangential component of the relative velocity of the two contacting points after impact is given as

$$\boldsymbol{u}'_\mathrm{t} = \varepsilon_\mathrm{t}\boldsymbol{u}_\mathrm{t}. \tag{11}$$

Using Eq. (11) and conservation of linear and angular momenta, the change of $\boldsymbol{v}_\mathrm{t}$ can be written as

$$\begin{aligned} \Delta\boldsymbol{v}_\mathrm{t} &= -\frac{\mathcal{K}}{\mathcal{K}+1}\Delta\boldsymbol{u}_\mathrm{t} \\ &= -\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\{\boldsymbol{v}_\mathrm{t} + R_\mathrm{p}\boldsymbol{\lambda} \times (\boldsymbol{\omega} - \boldsymbol{\Omega})\}, \end{aligned} \tag{12}$$

where $\Delta\boldsymbol{u}_\mathrm{t} \equiv \boldsymbol{u}'_\mathrm{t} - \boldsymbol{u}_\mathrm{t}$, $R_\mathrm{p} \equiv R_1 + R_2$, and

$$\boldsymbol{\omega} \equiv (R_1\boldsymbol{\omega}_1 + R_2\boldsymbol{\omega}_2)/(R_1 + R_2). \tag{13}$$

$\mathcal{K}$ is the coefficient for the moment of inertia, i.e., $I_1 = \mathcal{K}m_1R_1^2$, and $\mathcal{K} = 2/5$ for a homogeneous sphere.

Then, the change of the spin angular momentum of the moonlet can be written as

$$\begin{aligned} \Delta\boldsymbol{L}_1 &= I_1\Delta\boldsymbol{\omega}_1 = R_1\boldsymbol{\lambda} \times (-\mu\Delta\boldsymbol{v}) \\ &= \frac{\mathcal{K}}{\mathcal{K}+1}\mu(1-\varepsilon_\mathrm{t})R_1\boldsymbol{\lambda} \\ &\quad \times \{\boldsymbol{v}_\mathrm{t} + R_\mathrm{p}\boldsymbol{\lambda} \times (\boldsymbol{\omega} - \boldsymbol{\Omega})\}, \end{aligned} \tag{14}$$

where μ is the reduced mass. If we assume $m_1 \gg m_2$, then $\mu \simeq m_2$ and $R_1 \simeq R_\mathrm{p}$. In this case, the specific angular momentum delivered to the moonlet, $\boldsymbol{l} = \Delta\boldsymbol{L}/m_2$, can be written as

$$\begin{aligned} \boldsymbol{l} &\simeq \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\{\boldsymbol{l}^\mathrm{r} + R_\mathrm{p}^2\boldsymbol{\lambda} \times (\boldsymbol{\lambda} \times (\boldsymbol{\omega} - \boldsymbol{\Omega}))\} \\ &= \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\{\boldsymbol{l}^\mathrm{r} + R_\mathrm{p}^2(\boldsymbol{\lambda} \cdot (\boldsymbol{\omega} - \boldsymbol{\Omega}))\boldsymbol{\lambda} \\ &\quad - R_\mathrm{p}^2(\boldsymbol{\omega} - \boldsymbol{\Omega})\}, \end{aligned} \tag{15}$$

where $\boldsymbol{l}^\mathrm{r} = R_\mathrm{p}\boldsymbol{\lambda} \times \boldsymbol{v}_\mathrm{t}$ is the translational angular momentum of the particle's motion relative to the moonlet at impact, measured in the rotational coordinate system.

The rate of accumulation of the z-component of nondimensional specific angular momentum, $\tilde{l}_z = l_z/R_\mathrm{H}^2\Omega$, can be defined as

$$L_z \equiv \int \tilde{l}_z p_\mathrm{col}(\tilde{e}, \tilde{i}, \tilde{b}, \tau, \varpi)\frac{3}{2}|\tilde{b}|d\tilde{b}\frac{d\tau d\varpi}{(2\pi)^2}. \tag{16}$$

Using Eqs. (3) and (16), we obtain the z-component of the mean angular momentum accreted per unit mass of impacting particles as[1]

$$\langle l_z \rangle = R_\mathrm{H}^2\Omega L_z/P_\mathrm{col}. \tag{17}$$

Similarly, we define L_x, L_y, $\langle l_x \rangle$, and $\langle l_y \rangle$. Also, we define the rate of accumulation of the square of the total nondimensional angular momentum ($\tilde{l}^2 = l^2/(R_\mathrm{H}^2\Omega)^2$, where $l^2 = l_x^2 + l_y^2 + l_z^2$), and that of $\tilde{l}_z^2 = l_z^2/(R_\mathrm{H}^2\Omega)^2$, as

$$\begin{aligned} L_2 &\equiv \int \tilde{l}^2 p_\mathrm{col}(\tilde{e}, \tilde{i}, \tilde{b}, \tau, \varpi)\frac{3}{2}|\tilde{b}|d\tilde{b}\frac{d\tau d\varpi}{(2\pi)^2}, \\ L_{z2} &\equiv \int \tilde{l}_z^2 p_\mathrm{col}(\tilde{e}, \tilde{i}, \tilde{b}, \tau, \varpi)\frac{3}{2}|\tilde{b}|d\tilde{b}\frac{d\tau d\varpi}{(2\pi)^2}. \end{aligned} \tag{18}$$

[1] Note that the brackets in Eqs. (17) and (19) represent the ensemble average over the collision orbits with a given pair of $\tilde{e}$ and $\tilde{i}$. We will use the same expression to denote the Rayleigh distribution averages of these quantities ($\langle l_z \rangle$, $\langle l^2 \rangle^{1/2}$, and $\langle l_z^2 \rangle^{1/2}$; see Eqs. (34) and (48)), since we think that it is not confusing.

Then, the mean squares $\langle l^2\rangle$ and $\langle l_z^2\rangle$ can be given as

$$\langle l^2\rangle = (R_{\rm H}^2\Omega)^2 L_2/P_{\rm col},$$
$$\langle l_z^2\rangle = (R_{\rm H}^2\Omega)^2 L_{z2}/P_{\rm col}. \tag{19}$$

The Rayleigh distribution averages such as $\langle L_z\rangle$ and $\langle L_2\rangle$ can be also defined, in a similar manner to the case of $\langle P_{\rm col}\rangle$ defined by Eq. (7).

3. Analytic Calculation of the Mean and Mean Square Angular Momenta

3.1 Mean angular momentum

Each component of Eq. (15) can be written as

$$l_x = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t})\{l_x^t - (y^2+z^2)\omega_x + x(y\omega_y + z\omega_z)\},$$
$$l_y = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t})\{l_y^t - (z^2+x^2)\omega_y + y(z\omega_z + x\omega_x)\},$$
$$l_z = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t})\{l_z^t - (x^2+y^2)\omega_z + z(x\omega_x + y\omega_y)\}, \tag{20}$$

where

$$l_x^t = y\dot z - z\dot y - \Omega xz$$
$$l_y^t = z\dot x - x\dot z - \Omega yz \tag{21}$$
$$l_z^t = x\dot y - y\dot x + \Omega(x^2+y^2)$$

are the components of the translational angular momentum of the impacting particle in the inertial coordinate system centered on the moonlet (e.g., Lissauer and Kary, 1991).

The mean angular momentum $\langle l_x\rangle$, $\langle l_y\rangle$, and $\langle l_z\rangle$ in the three-dimensional case with non-zero orbital inclinations can be evaluated analytically when (i) $\tilde e, \tilde i \gg \tilde v_{\rm e}$ ($\tilde v_{\rm e} \equiv v_{\rm e}/R_{\rm H}\Omega$ is the scaled escape velocity) and (ii) $\tilde e, \tilde i \gg \tilde r_{\rm p}$. In this case, the relative velocity at $(\tilde x, \tilde y, \tilde z) = (0,0,0)$ in the non-gravitating solution to Eqs. (1) is given by (Nakazawa *et al.*, 1989)

$$(\dot{\tilde x}, \dot{\tilde y}, \dot{\tilde z}) = \left(\pm\left(\tilde e^2 - \tilde b^2\right)^{1/2}, \tilde b/2, \pm\tilde i\right), \tag{22}$$

where each of the four combinations of the signs of $\dot{\tilde x}$ and $\dot{\tilde z}$ respectively correspond to each of the four bands of collision orbits for given $\tilde e$, $\tilde i$, and and $\tilde b$. Owing to the symmetry among these four collision bands, the last terms in Eqs. (20) cancel out. Also, $\langle l_x^t\rangle = \langle l_y^t\rangle = 0$, by symmetry with respect to the $z = 0$ plane. These characteristics simplify the derivation of analytic expressions of the mean angular momentum.

First, we derive the analytic expression for L_z defined by Eq. (16). Here, we define

$$L_{tz} \equiv \int \tilde l_z^t p_{\rm col}(\tilde e, \tilde i, \tilde b, \tau, \varpi)\frac{3}{2}|\tilde b| d\tilde b \frac{d\tau d\varpi}{(2\pi)^2},$$
$$L_{x^2+y^2} \equiv \int (\tilde x^2 + \tilde y^2) p_{\rm col}(\tilde e, \tilde i, \tilde b, \tau, \varpi)\frac{3}{2}|\tilde b| d\tilde b \frac{d\tau d\varpi}{(2\pi)^2}, \tag{23}$$

where $\tilde l_z^t = l_z^t/(R_{\rm H}^2\Omega)$ and $\tilde x^2 + \tilde y^2 = (x^2+y^2)/R_{\rm H}^2$. From Eqs. (16) and (20), L_z can be written in terms of these quantities as

$$L_z = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t})(L_{tz} - L_{x^2+y^2}\cdot\omega_z/\Omega). \tag{24}$$

The analytic expression for L_{tz} is obtained by Lissauer *et al.* (1997) as

$$L_{tz} = \frac{\tilde r_{\rm p}^4\left(\tilde e^2 + \tilde i^2\right)^{1/2}}{\pi\tilde i} \times \left(E(k) - \frac{\tilde e^2 + 3\tilde i^2}{4\left(\tilde e^2 + \tilde i^2\right)}K(k) + \frac{1}{12\pi}\right), \tag{25}$$

where $K(k)$ is the complete elliptic integral of the first kind. Note that the last term on the r.h.s. of Eq. (25) accounts for self-shadowing by the moonlet for those orbits with small $\tilde b$ ($< 2\tilde r_{\rm p}/3\pi$) where the motion of the guiding center of a particle during one epicyclic period is less than the moonlet's diameter (Lissauer and Kary, 1991; Dones and Tremaine, 1993b). This correction needs to be taken into account for the evaluation of L_{tz} because of near cancellation of the contributions of the positive and negative angular momenta in the bulk region with $2\tilde r_{\rm p}/3\pi < \tilde b \le \tilde e$. On the other hand, such a correction is not necessary for $P_{\rm col}$, $L_{x^2+y^2}$, L_2, and L_{z2}, where such cancellation does not occur. In order to obtain the analytic expression for $L_{x^2+y^2}$, we introduce a new coordinate system $(\tilde x', \tilde y', \tilde z')$ defined as (Ohtsuki, 1992, 1999)

$$\begin{pmatrix}\tilde x\\ \tilde y\\ \tilde z\end{pmatrix} = \begin{pmatrix}\sin\theta_0 & \cos\theta_0 & 0\\ -\cos\theta_0 & \sin\theta_0 & 0\\ 0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}\sin\theta_1 & 0 & \cos\theta_1\\ 0 & 1 & 0\\ -\cos\theta_1 & 0 & \sin\theta_1\end{pmatrix}\begin{pmatrix}\tilde x'\\ \tilde y'\\ \tilde z'\end{pmatrix} \tag{26}$$

with

$$\cos\theta_0 \equiv \frac{\tilde b}{2}\left(\tilde e^2 - \frac{3}{4}\tilde b^2\right)^{-1/2},$$
$$\sin\theta_0 \equiv \left(\tilde e^2 - \tilde b^2\right)^{1/2}\left(\tilde e^2 - \frac{3}{4}\tilde b^2\right)^{-1/2},$$
$$\cos\theta_1 \equiv \tilde i\left(\tilde e^2 + \tilde i^2 - \frac{3}{4}\tilde b^2\right)^{-1/2}, \tag{27}$$
$$\sin\theta_1 \equiv \left(\tilde e^2 - \frac{3}{4}\tilde b^2\right)^{1/2}\left(\tilde e^2 + \tilde i^2 - \frac{3}{4}\tilde b^2\right)^{-1/2}.$$

Note that the magnitude of each component of the impact velocity under the present approximation is $\left(\left(\tilde e^2 - \tilde b^2\right)^{1/2}, \tilde b/2, \tilde i\right)$ (Eq. (22)). The magnitude of the impact velocity is $\left(\tilde e^2 + \tilde i^2 - 3\tilde b^2/4\right)^{1/2}$, while the magnitude of the impact velocity projected onto the $\tilde z = 0$ plane is $\left(\tilde e^2 - 3\tilde b^2/4\right)^{1/2}$. Therefore, θ_0 in the above equations represents the direction of an incoming particle in the $\tilde x$-$\tilde y$ plane, while θ_1 denotes the angle between the direction of the incoming particle and the $\tilde z$-axis. Using Eqs. (26) and (27), we can express $\tilde{x}^2 + \tilde{y}^2$ in terms of $(\tilde{x}', \tilde{y}', \tilde{z}')$. Taking averages of $\tilde{x}^2 + \tilde{y}^2$ over $\tilde{y}'$ and $\tilde{z}'$ (which corresponds to averaging over orbital phases) and integrating over $\tilde{b}$ together with the collision probability $p_{\rm col}(\tilde{e}, \tilde{i}, \tilde{b})$ given by Eq. (4), we obtain

$$L_{x^2+y^2} = \frac{\tilde{r}_{\rm p}^4 \left(\tilde{e}^2 + \tilde{i}^2\right)^{1/2}}{\pi \tilde{i}} \left(\frac{3}{2} E(k) - \frac{\tilde{i}^2}{2\left(\tilde{e}^2 + \tilde{i}^2\right)} K(k) \right). \tag{28}$$

From Eqs. (24), (25), and (28), we obtain

$$\begin{aligned} L_z = \; & \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) \frac{\tilde{r}_{\rm p}^4 \left(\tilde{e}^2 + \tilde{i}^2\right)^{1/2}}{\pi \tilde{i}} \\ & \times \left\{ \left(E(k) - \frac{\tilde{e}^2 + 3\tilde{i}^2}{4\left(\tilde{e}^2 + \tilde{i}^2\right)} K(k) + \frac{1}{12\pi} \right) \right. \\ & \left. - \left(\frac{3}{2} E(k) - \frac{\tilde{i}^2}{2\left(\tilde{e}^2 + \tilde{i}^2\right)} K(k) \right) \frac{\omega_z}{\Omega} \right\}. \end{aligned} \tag{29}$$

Using Eqs. (5) and (29), we obtain the expression for $\langle l_z \rangle$ for the case where all the particles have identical eccentricities and inclinations, as

$$\langle l_z \rangle = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) \left(\langle l_z^t \rangle - \langle x^2 + y^2 \rangle \omega_z \right), \tag{30}$$

where

$$\begin{aligned} \langle l_z^t \rangle &= R_{\rm p}^2 \Omega \left\{ \frac{1}{2} - \frac{\tilde{e}^2 + 3\tilde{i}^2}{8\left(\tilde{e}^2 + \tilde{i}^2\right)} \frac{K(k)}{E(k)} + \frac{1}{24\pi E(k)} \right\}, \\ \langle x^2 + y^2 \rangle &= R_{\rm p}^2 \left\{ \frac{3}{4} - \frac{\tilde{i}^2}{4\left(\tilde{e}^2 + \tilde{i}^2\right)} \frac{K(k)}{E(k)} \right\}. \end{aligned} \tag{31}$$

Equation (30) can be also written in the form $\langle l_z \rangle \propto \omega_{\rm eq} - \omega_z$, where $\omega_{\rm eq} = \langle l_z^t \rangle / \langle x^2 + y^2 \rangle$, which suggests the existence of an equilibrium rotation for which $\langle l_z \rangle$ vanishes. We have $\omega_{\rm eq} = 0.3665\Omega$ for $e = 2i$.

Following Ohtsuki (1999), who calculated the Rayleigh distribution averages of the viscous stirring and dynamical friction rates of ring particles, we can evaluate the Rayleigh distribution average of L_z as

$$\langle L_z \rangle = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) \frac{\tilde{r}_{\rm p}^4}{\pi} \left\{ I_{tz}(\beta) - I_{x^2+y^2}(\beta)(\omega_z/\Omega) \right\}, \tag{32}$$

where

$$I_{tz}(\beta) \equiv \int_0^1 \frac{2E\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right) - \left(\frac{1}{2} + \chi^2\right) K\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right) + \frac{1}{6\pi}}{\left\{\beta + (\beta^{-1} - \beta)\chi^2\right\}^2} d\chi,$$

$$I_{x^2+y^2}(\beta) \equiv \int_0^1 \frac{3E\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right) - \chi^2 K\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)}{\left\{\beta + (\beta^{-1} - \beta)\chi^2\right\}^2} d\chi. \tag{33}$$

We show the plots of $I_{tz}(\beta)$ and $I_{x^2+y^2}(\beta)$ in Fig. 1. We derived approximate analytic expressions for these quantities in Appendix, and are also plotted in Fig. 1 with the open circles. Using Eqs. (8) and (32), we obtain the analytic expression of $\langle l_z \rangle$ in the case with the Rayleigh distribution of particles' eccentricities and inclinations as

$$\langle l_z \rangle = C_z(\beta) \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) R_{\rm p}^2 \left(\omega_{\rm eq} - \omega_z\right), \tag{34}$$

where

$$\begin{aligned} C_z(\beta) &\equiv I_{x^2+y^2}(\beta)/I(\beta), \\ \omega_{\rm eq} &\equiv \left\{ I_{tz}(\beta)/I_{x^2+y^2}(\beta) \right\} \Omega. \end{aligned} \tag{35}$$

When $\beta = 1/2$, we have $C_z = 0.6998$ and $\omega_{\rm eq} = 0.3712\Omega$. The above value of the equilibrium rotation rate is slightly larger than the results of previous analytic studies on rotating ring particles with identical sizes (Salo, 1987a; Araki, 1991), which gave $\omega_{\rm eq} \sim 0.3\Omega$. In the limit of $\beta \ll 1$, all of I, I_{tz}, and $I_{x^2+y^2}$ are proportional to β^{-1} (Eqs. (A.8) and (A.14)). As a result, both C_z and $\omega_{\rm eq}$ become independent of β in this limit, and we have $C_z = 0.75$ and $\omega_{\rm eq} = 0.3837\Omega$. Note that $\beta \simeq 1/2$ is expected in such high-velocities that $\langle \tilde{e}^2 \rangle^{1/2} \gtrsim 2$, where the random velocity dominates particles' relative velocity. On the other hand, β can take on smaller values if the velocity stirring of particles occurs in the lower-velocity regime with $\langle \tilde{e}^2 \rangle^{1/2} \lesssim 1$, where Kepler shear dominates the relative velocity (Ohtsuki, 1999).

Analytic expressions of the x- and the y-components of the mean angular momentum can be also derived in a similar manner. We obtain

$$\begin{aligned} L_x =& -\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) \frac{\tilde{r}_{\rm p}^4 \left(\tilde{e}^2 + \tilde{i}^2\right)^{1/2}}{\pi \tilde{i}} \\ & \times \left\{ \frac{5}{6} E(k) + \frac{\tilde{e}^2 + 4\tilde{i}^2}{6\left(\tilde{e}^2 + \tilde{i}^2\right)} K(k) \right\} \frac{\omega_x}{\Omega}, \\ L_y =& -\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) \frac{\tilde{r}_{\rm p}^4 \left(\tilde{e}^2 + \tilde{i}^2\right)^{1/2}}{\pi \tilde{i}} \\ & \times \left\{ \frac{5}{3} E(k) - \frac{1}{6} K(k) \right\} \frac{\omega_y}{\Omega}, \end{aligned} \tag{36}$$

and their Rayleigh distribution averages are given by

$$\langle L_x\rangle = -\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\frac{\tilde{r}_\mathrm{p}^4}{\pi}I_{y^2+z^2}(\beta)\omega_x/\Omega,$$
$$\langle L_y\rangle = -\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\frac{\tilde{r}_\mathrm{p}^4}{\pi}I_{z^2+x^2}(\beta)\omega_y/\Omega, \tag{37}$$

where

$$I_{y^2+z^2}(\beta) \equiv \frac{1}{3}\int_0^1 \frac{5E\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)+\left(1+3\chi^2\right)K\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)}{\left\{\beta+(\beta^{-1}-\beta)\chi^2\right\}^2}d\chi,$$

$$I_{z^2+x^2}(\beta) \equiv \frac{1}{3}\int_0^1 \frac{10E\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)-K\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)}{\left\{\beta+(\beta^{-1}-\beta)\chi^2\right\}^2}d\chi. \tag{38}$$

We show the plots of $I_{y^2+z^2}(\beta)$ and $I_{z^2+x^2}(\beta)$ in Fig. 1 (solid lines), together with their values calculated by the approximate analytic expressions derived in Appendix (open circles).

Using Eqs. (8) and (37), we obtain

$$\langle l_x\rangle = -C_x(\beta)\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})R_\mathrm{p}^2\omega_x,$$
$$\langle l_y\rangle = -C_y(\beta)\frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})R_\mathrm{p}^2\omega_y, \tag{39}$$

where

$$C_x(\beta) \equiv I_{y^2+z^2}(\beta)/I(\beta),$$
$$C_y(\beta) \equiv I_{z^2+x^2}(\beta)/I(\beta). \tag{40}$$

Note that the magnitude of the x- and the y-components of the moonlet's rotation rate defined in the above rotating coordinate system changes with time even without particle collisions (Morishima and Salo, 2004; Ohtsuki, 2004), and Eqs. (39) represent the values for the instantaneous directions of the x- and the y-axes. When $\beta = 1/2$, we have $C_x = 0.6005$ and $C_y = 0.6997$. In the limit of $\beta \ll 1$, both C_x and C_y become independent of β, and we have $C_x = 0.5654$ and $C_y = 0.6846$.

Equations (34) and (39) show that the mean angular momentum of a large number of impacts tends to lead to the equilibrium rotation with $\boldsymbol{\omega}_\mathrm{eq} = (0, 0, \omega_\mathrm{eq})$.

3.2 Mean square angular momentum

The mean square angular momenta $\langle l^2\rangle$ and $\langle l_z^2\rangle$ can be evaluated analytically when the above conditions, (i) $\tilde{e}, \tilde{i} \gg \tilde{v}_\mathrm{e}$ and (ii) $\tilde{e}, \tilde{i} \gg \tilde{r}_\mathrm{p}$, are satisfied. From the result described in the last subsection, we can expect that $\mathcal{O}(|\boldsymbol{\omega}|) \lesssim \Omega$. In this case, the first and the second terms in the braces on the r.h.s. of Eq. (15) are on the order of $R_\mathrm{p} e v_\mathrm{K}$ (v_K is the circular Keplerian velocity of the moonlet) and $R_\mathrm{p}^2\Omega$, respectively. Therefore, the second term can be neglected compared to the first term, since $R_\mathrm{p}\Omega/ev_\mathrm{K} \ll 1$ under the above assumption (ii). Then, the three components of $\boldsymbol{l}$ given as Eq. (15) scaled by $R_\mathrm{H}^2\Omega$ can be written in terms of the scaled relative position $(\tilde{x}, \tilde{y}, \tilde{z})$ and the scaled relative velocity $(\dot{\tilde{x}}, \dot{\tilde{y}}, \dot{\tilde{z}})$ at impact, as

$$\tilde{l}_x = l_x/R_\mathrm{H}^2\Omega = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\left(\tilde{y}\dot{\tilde{z}} - \tilde{z}\dot{\tilde{y}}\right),$$
$$\tilde{l}_y = l_y/R_\mathrm{H}^2\Omega = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\left(\tilde{z}\dot{\tilde{x}} - \tilde{x}\dot{\tilde{z}}\right),$$
$$\tilde{l}_z = l_z/R_\mathrm{H}^2\Omega = \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_\mathrm{t})\left(\tilde{x}\dot{\tilde{y}} - \tilde{y}\dot{\tilde{x}}\right). \tag{41}$$

Substituting Eqs. (22), (26), and (27) into Eq. (41), we get

$$\tilde{l}^2 = \left(\frac{\mathcal{K}}{\mathcal{K}+1}\right)^2(1-\varepsilon_\mathrm{t})^2\left(\tilde{e}^2+\tilde{i}^2-\frac{3}{4}\tilde{b}^2\right)\left(\tilde{y}'^2+\tilde{z}'^2\right),$$
$$\tilde{l}_z^2 = \left(\frac{\mathcal{K}}{\mathcal{K}+1}\right)^2(1-\varepsilon_\mathrm{t})^2\left(\tilde{e}^2-\frac{3}{4}\tilde{b}^2\right)\tilde{y}'^2. \tag{42}$$

Taking averages of $\tilde{l}^2$ over $\tilde{y}'$ and $\tilde{z}'$ and integrating over $\tilde{b}$ together with the collision probability $p_\mathrm{col}(\tilde{e}, \tilde{i}, \tilde{b})$ given by Eq. (4), we obtain the expression for the nondimensional accretion rate of mean square angular momentum L_2 defined by Eq. (18) as

$$L_2 = \left(\frac{\mathcal{K}}{\mathcal{K}+1}\right)^2(1-\varepsilon_\mathrm{t})^2\frac{\tilde{r}_\mathrm{p}^4\left(\tilde{e}^2+\tilde{i}^2\right)^{3/2}}{\pi\tilde{i}}\left(E(k)-k^2F(k)\right), \tag{43}$$

where $F(k)$ is defined in terms of $E(k)$ and $K(k)$ as

$$F(k) \equiv \frac{1}{3k^2}\left[(2k^2-1)E(k)+(1-k^2)K(k)\right]. \tag{44}$$

Similarly, we obtain

$$L_{z2} = \left(\frac{\mathcal{K}}{\mathcal{K}+1}\right)^2(1-\varepsilon_\mathrm{t})^2 \times \frac{\tilde{r}_\mathrm{p}^4\tilde{e}^2\left(\tilde{e}^2+\tilde{i}^2\right)^{1/2}}{2\pi\tilde{i}}\left(E(k)-\frac{3}{4}F(k)\right). \tag{45}$$

The Rayleigh distribution averages of these quantities can be evaluated as

$$\langle L_2\rangle = \left(\frac{\mathcal{K}}{\mathcal{K}+1}\right)^2(1-\varepsilon_\mathrm{t})^2\frac{I_2(\beta)\tilde{r}_\mathrm{p}^4}{\pi}\langle\tilde{e}^2\rangle,$$
$$\langle L_{z2}\rangle = \left(\frac{\mathcal{K}}{\mathcal{K}+1}\right)^2(1-\varepsilon_\mathrm{t})^2\frac{I_{z2}(\beta)\tilde{r}_\mathrm{p}^4}{\pi}\langle\tilde{e}^2\rangle, \tag{46}$$

where

$$
\begin{aligned}
I_2(\beta) \equiv & \int_0^1 \\
& \frac{4E\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right) - 3(1-\chi^2)F\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)}{\beta^{-1}\left\{\beta + (\beta^{-1}-\beta)\chi^2\right\}^3} d\chi, \\
I_{z2}(\beta) \equiv & \int_0^1 \\
& \frac{(1-\chi^2)\left\{2E\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right) - \frac{3}{2}F\left(\frac{\sqrt{3}}{2}\sqrt{1-\chi^2}\right)\right\}}{\beta^{-1}\left\{\beta + (\beta^{-1}-\beta)\chi^2\right\}^3} d\chi.
\end{aligned} \tag{47}
$$

The plots of $I_2(\beta)$ and $I_{z2}(\beta)$ are shown in Fig. 1. We derived approximate analytic expressions for these integrals in Appendix, and are plotted in Fig. 1 with the open circles. From Eqs. (8) and (46), we obtain

$$
\begin{aligned}
\langle l^2 \rangle^{1/2} &= R_{\rm H}^2 \Omega \left(\langle L_2 \rangle / \langle P_{\rm col} \rangle\right)^{1/2} \\
&= C_2(\beta) \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) R_{\rm p} \sigma_x, \\
\langle l_z^2 \rangle^{1/2} &= R_{\rm H}^2 \Omega \left(\langle L_{z2} \rangle / \langle P_{\rm col} \rangle\right)^{1/2} \\
&= C_{z2}(\beta) \frac{\mathcal{K}}{\mathcal{K}+1}(1-\varepsilon_{\rm t}) R_{\rm p} \sigma_x,
\end{aligned} \tag{48}
$$

where $\sigma_x \equiv (\langle e^2 \rangle/2)^{1/2} a\Omega$ and

$$
\begin{aligned}
C_2(\beta) &\equiv \{I_2(\beta)/I(\beta)\}^{1/2}, \\
C_{z2}(\beta) &\equiv \{I_{z2}(\beta)/I(\beta)\}^{1/2}.
\end{aligned} \tag{49}
$$

When $\beta = 1/2$, we have $C_2 = 1.048$ and $C_{z2} = 0.6889$. In the limit of $\beta \ll 1$, both C_2 and C_{z2} become independent of β, and we have $C_2 = 1.013$ and $C_{z2} = 0.7165$.

4. Systematic and Random Components of Moonlet Rotation

The result for the mean angular momentum obtained in Section 3.1 (Eq. (34)) shows that the moonlet would achieve the equilibrium rotation rate in the prograde direction with $\boldsymbol{\omega}_{\rm eq} = (0, 0, \omega_{\rm eq})$ after experiencing a large number of small impacts, where $\omega_{\rm eq}$ is given by Eq. (35). However, in addition to the above systematic components of rotation, the random component imparted by large impacts also needs to be taken into account, as we mentioned above. Dones and Tremaine (1993a, b) derived a unified formulation for the two components for planetary rotation under the assumption that the planet accretes planetesimals whenever they collide. In this case, the mass of the planet is equal to the total mass of accreted planetesimals. On the other hand, in the case of moonlet rotation in non-gravitating case that we consider in the present work, particles rebound after impacts, and the mass of the moonlet is kept constant. In the following, we derive an alternative formulation that allows a unified analysis of the systematic and the random components of moonlet rotation. We also discuss the relative importance of these two components using the results obtained in Section 3.

Suppose that the initial values of the spin angular momentum and the spin angular velocity vector of a moonlet (mass M, radius R, moment of inertia $I = \mathcal{K} M R^2$) are given by $\boldsymbol{L}_0$ and $\boldsymbol{\omega}_0$ $(= \boldsymbol{L}_0/I)$, respectively. The spin angular momentum that the moonlet acquires after N impacts of particles can be written as

$$
\boldsymbol{L} = \sum_{i=1}^{N} m_i \boldsymbol{l}_i, \tag{50}
$$

where $\boldsymbol{l}_i$ is the specific angular momentum delivered by the i-th impactor m_i. Then the expectation value of $\boldsymbol{L}$ can be written as

$$
\langle \boldsymbol{L} \rangle = N \langle m \rangle \langle \boldsymbol{l} \rangle, \tag{51}
$$

where $\langle \boldsymbol{l} \rangle \equiv (\langle l_x \rangle, \langle l_y \rangle, \langle l_z \rangle)$ are given by Eqs. (34) and (39). The expectation value of the i-component ($i = x, y, z$) of the final rotation rate of the moonlet, $\langle \omega_{{\rm f},i} \rangle$, can be given by

$$
\langle \omega_{{\rm f},i} \rangle = \omega_{0,i} + N \langle m \rangle \langle l_i \rangle / I. \tag{52}
$$

From Eqs. (34), (39), (51), and (52), we find that an equilibrium state with $\langle \boldsymbol{\omega}_{\rm f} \rangle \sim (0, 0, \omega_{\rm eq})$ and $\langle \boldsymbol{L}_{\rm f} \rangle \sim \boldsymbol{L}_{\rm eq}$ $(= I \boldsymbol{\omega}_{\rm eq}$, where $\boldsymbol{\omega}_{\rm eq} = (0, 0, \omega_{\rm eq}))$ would be achieved after N impacts of particles if the random component can be neglected, where N satisfies the following relation:

$$
N \langle m \rangle (1 - \varepsilon_{\rm t}) \sim \frac{\mathcal{K}+1}{C_i} M \sim M, \tag{53}
$$

where $C_i \sim 0.6 - 0.7$ for $i = x, y, z$ (Eqs. (35) and (40)) and we assume $\mathcal{K} = 2/5$. The above N can be regarded as the number of impacts required to erase the memory of the moonlet's initial rotation. On the other hand, the expectation value of $\boldsymbol{L}_{\rm f}^2$ after the above N impacts can be written as

$$
\langle \boldsymbol{L}_{\rm f}^2 \rangle = \langle (\boldsymbol{L}_0 + \boldsymbol{L})^2 \rangle \sim \boldsymbol{L}_{\rm eq}^2 + N \langle m^2 \rangle \langle l^2 \rangle, \tag{54}
$$

where we assumed $N \gg 1$ and used the relation $\langle \boldsymbol{L}^2 \rangle \sim \langle \boldsymbol{L} \rangle^2 + N \langle m^2 \rangle \langle l^2 \rangle$ (Dones and Tremaine, 1993b; Ohtsuki and Ida, 1998). The first term in the last equation in Eq. (54) represents the systematic component that tends to lead the moonlet rotation toward the equilibrium state with $\langle \boldsymbol{\omega}_{\rm f} \rangle \sim \boldsymbol{\omega}_{\rm eq}$, while the second term accounts for the contribution of the random component. Using Eqs. (52) and (53), we can rewrite Eq. (54) as

$$
\langle \boldsymbol{L}_{\rm f}^2 \rangle \sim \boldsymbol{L}_{\rm eq}^2 \left\{1 + S_{\rm m}^2 S_{\rm l}^2\right\} \tag{55}
$$

with

$$
S_{\rm m}^2 \equiv \frac{\langle m^2 \rangle}{M \langle m \rangle} \tag{56}
$$

and

$$
\begin{aligned}
S_{\rm l}^2 &\equiv \frac{\langle l^2 \rangle}{(\mathcal{K} R^2 \omega_{\rm eq})^2} \cdot \frac{\mathcal{K} R^2 (\omega_{\rm eq} - \omega_{0,z})}{\langle l_z \rangle} \\
&= C_{\rm l} \frac{(1-\varepsilon_{\rm t})\sigma_x^2}{R^2 \omega_{\rm eq}^2},
\end{aligned} \tag{57}
$$

where we used Eqs. (34) and (48), and $C_1 \equiv C_2^2/\{C_z(\mathcal{K}+1)\}$. C_z and C_2 are defined in Eqs. (35) and (49), respectively, and $C_1 = 1.122$ when $\beta = 1/2$ and $\mathcal{K} = 2/5$. Note that the final expression of S_l^2 does not depend on the moonlet's initial rotation rate ($\omega_{0,z}$), as $\langle l_z \rangle \propto (\omega_{eq} - \omega_z)$ (Eq. (34)). Equation (55) shows that the systematic component dominates the moonlet rotation when $S_m S_l < 1$, while the contribution of the random component can be significant if $S_m S_l \gtrsim 1$. Dones and Tremaine (1993b) obtained a relation similar to Eq. (55) for planetary rotation. In their formulation, the random component dominates planetary rotation when $S_{m,DT} S_{l,DT} > 1$, where $S_{m,DT}^2 = \langle m^2 \rangle/(M\langle m \rangle)$ and $S_{l,DT}^2 = \langle l^2 \rangle/\langle l_z \rangle^2$. In these expressions defined by Dones and Tremaine (1993b), $\langle m \rangle$ and $\langle m^2 \rangle$ are the mean and mean square masses of accreted planetesimals, and M in their case is the mass of the planet and is given by $M = N\langle m \rangle$. On the other hand, M in Eq. (56) is independent of the mass distribution of impacting particles and is assumed to be kept constant in the present work. Also, $S_{l,DT}^2$ is expressed only in terms of the angular momentum distribution of accreted planetesimals, while S_l^2 in Eq. (57) depends also on the size and rotation rate of the moonlet (Note that $\mathcal{K}R^2\omega$ represents the moonlet's spin angular momentum per unit mass).

If all the impacting particles have the same mass m, we have

$$S_m = (m/M)^{1/2}. \tag{58}$$

In a more realistic case where impacting particles have a size distribution given by $N(R)dR \propto R^{-q}dR$ with $q \sim 3$ for $m_0 \leq m \leq m_1$ (m_0 and m_1 are the mass of the smallest and the largest particles), we find

$$S_m \sim 0.5 \times \left(\frac{m_1}{M}\right)^{1/2}. \tag{59}$$

On the other hand, Eq. (57) shows that the relative importance of the systematic and the random components also depends on the values of ε_t and the magnitude of particles' velocity dispersion relative to the moonlet's equilibrium rotation velocity $R\omega_{eq}$, which is on the order of the Kepler shear across the moonlet's radius ($\sim R\Omega$), as $\mathcal{O}(\omega_{eq}) \sim \Omega$ (Eq. (35)).

In the above, we assumed that the mass of an impacting particle is much smaller than the moonlet's mass, and we replaced μ and R_1 in Eq. (14) with m_2 and R_p ($= R_1 + R_2$), respectively. When the particle mass (m_i) is not negligible compared to the moonlet's mass (M), each term of the r.h.s. of Eq. (50) needs to be multiplied by a factor

$$f(\mu_i') \equiv (1 + \mu_i')^{-1}(1 + \mu_i'^{1/3})^{-1}, \tag{60}$$

where $\mu_i' = m_i/M$.

In planetary rings with low optical depth, particles' velocity dispersion can be approximated by $\sigma_x \sim \max\{v_e, (2-3)R_2\Omega\}$ in the gravitating case and $\sigma_x \sim (2-3)R_2\Omega$ in the non-gravitating case (v_e and R_2 are the escape velocity and the radius of a particle), if the moonlet is small enough to neglect its perturbation on particles' velocities (e.g., Salo, 1995; Ohtsuki, 1999). Using Eq. (35), we then find that $S_l \sim (2-5)\,(m/M)^{1/3}$ in non-gravitating rings of equal-sized particles with $\varepsilon_t \sim 0.6 - 0.9$ and $m \ll M$. Thus, taking into account the correction factor $f(\mu')$ ($\mu' = m/M$), we obtain

$$S_m S_l \sim (2-5) \times \left(f(\mu')\right)^{1/2} \left(\frac{m}{M}\right)^{5/6}. \tag{61}$$

Figure 2 shows the plots of $S_m S_l$ in the non-gravitating, equal-sized-particle case with $\varepsilon_t = 0.7$ and $\sigma_x = 2R_2\Omega$, as a function of m/M. The above estimate shows that $S_m S_l < 1$ and the systematic component dominates the moonlet rotation as long as the mass of the largest impactors is much smaller than the moonlet's mass. In this case, $\langle \boldsymbol{\omega}_f \rangle \sim \boldsymbol{\omega}_{eq}$, and a slow prograde rotation of the moonlet can be expected. On the other hand, the random component can be significant, if the mass of impacting particles is comparable to the moonlet's mass and/or the velocity dispersion of particles is much larger than the Kepler shear across the moonlet's radius (e.g., by external perturbation). In this case, we find

$$\frac{\langle \omega_f^2 \rangle^{1/2}}{\Omega} \sim S_m S'_l, \tag{62}$$

where $S_l'^2 \equiv C_l(1-\varepsilon_t)\sigma_x^2/(R^2\Omega^2) = (\omega_{eq}^2/\Omega^2)S_l^2$. Equation (62) shows that the moonlet can have both prograde and retrograde rotations, and that the rotation rate would depend on the values of the mass of the largest impactors relative to the moonlet's mass (S_m) and the velocity dispersion of the impactors relative to the Kepler shear across the moonlet's radius (S_l').

5. Comparison with N-body Simulation

In order to confirm the validity of the formulation for moonlet rotation derived in Section 4 and the analytic results described in Section 3, we performed a local N-body simulation for a system of a moonlet embedded in planetary rings, neglecting their mutual gravity. The numerical method we

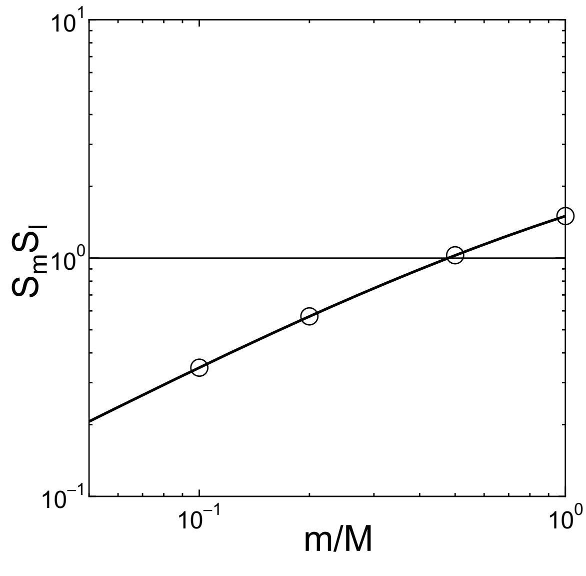

Fig. 2. The values of $S_m S_l$ defined by Eqs. (56) and (57) in non-gravitating rings of equal-sized particles with $\varepsilon_t = 0.7$ are shown as a function of m/M, where m and M are the mass of a particle and the moonlet, respectively. The open circles show the values of m/M for which N-body simulation was performed in Fig. 3 (see Section 5).

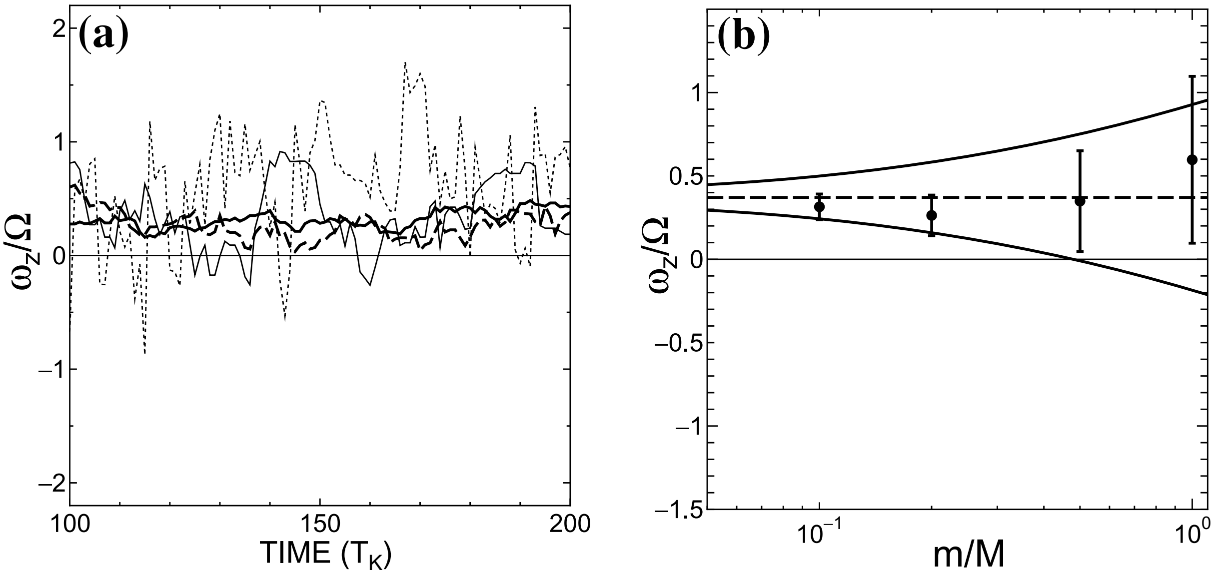

Fig. 3. (a) Numerical results of N-body simulation for the evolution of the z-component of the rotation rate (scaled by the Keplerian orbital frequency Ω) of a non-gravitating moonlet embedded in planetary rings of equal-sized particles. Four cases with different particle-moonlet mass ratios are shown; $m/M = 0.1$ (thick solid line), 0.2 (thick dashed line), 0.5 (thin solid line), and 1 (thin dotted line). (b) Mean value and dispersions of the z-component of the moonlet's rotation rate. The thin dashed and the thin solid lines represent the equilibrium rotation rate and the dispersion obtained by the analytic calculations, while the solid circles and the error bars show the mean values and the dispersions obtained from the results of N-body simulation.

used is similar to our previous one described in Ohtsuki and Emori (2000).

Figure 3(a) shows the evolution of the z-component of the moonlet's rotation rate ($T_{\rm K}$ is the orbital period). In this simulation, one moonlet is embedded in a swarm of one thousand ring particles with their optical depth of 0.1, and their restitution coefficients in normal and tangential directions are 0.5 and 0.7, respectively. Four lines show the results with different particle-moonlet mass ratios ($m/M =$ 0.1, 0.2, 0.5, and 1). We show the plots for $100T_{\rm K} \le t \le 200T_{\rm K}$ to demonstrate the quasi-equilibrium behavior of the system. When the mass ratio is small ($0.1 - 0.2$), we find that the contribution of the random component is negligible or relatively small, and the values of ω_z show only slight fluctuation around its equilibrium value of $\sim (0.3 - 0.4) \times \Omega$. On the other hand, when the mass of the particle is comparable to that of the moonlet (i.e., $m/M \gtrsim 0.5$), the random component becomes significant and the rotation rate shows large fluctuation. In this case, it takes on even negative values (i.e., retrograde rotation). These results are consistent with the above analytic prediction (Fig. 2) based on our new formulation.

In order to further compare the analytic and the N-body results quantitatively, we calculated the dispersion of the moonlet's rotation rate in both cases. In the case of the analytic calculations, the dispersion is given by Eq. (62). On the other hand, in the case of the N-body simulation, we calculated the mean values and the dispersions directly from the numerical results shown in Fig. 3(a). Figure 3(b) shows the plots of the dispersions around the mean value as a function of m/M. The horizontal dashed line and the two solid lines represent the mean value and the dispersion calculated by the analytic results. The solid circles and the vertical lines show the mean values and the dispersions obtained by N-body simulation. These results show quite good agreement between the analytic and the N-body results.

6. Conclusions and Discussion

In the present work, we have evaluated analytically the mean and mean square spin angular momenta that a moonlet acquires by inelastic collisions of ring particles, taking account of the Rayleigh distribution of particles' eccentricities and inclinations. Our results for the three components of the mean angular momentum presented in Section 3 suggest an equilibrium rotation of a moonlet in the prograde direction that is slower than the synchronous rotation. However, in addition to the above systematic component arising from an average of the angular momentum provided by a number of small impacts, the random component that comes from large impacts also needs to be taken into account. We have derived a new formulation that allows a unified treatment of the systematic and the random components of moonlet rotation. In the non-gravitating case, we have found that the contribution of the random component would be significant when the mass of largest impactors is comparable to the moonlet's mass and/or the velocity dispersion of particles is larger than the Kepler shear across the moonlet's radius. We also confirmed quantitative agreement between the analytic results and those obtained by N-body simulation. On the other hand, the assumption that a moonlet is on a circular orbit is not a good approximation when the mass of an impacting particle is comparable to the moonlet's mass. We also have to consider energy exchange between rotation and random motion (Salo, 1987b). All these effects become important for rotation of ring particles with a continuous size distribution. The observation of thermal emission from Saturn's rings by the Voyager spacecraft suggests slow rotation of ring particles (Hanel *et al.*, 1981). Since it is expected that the composite infrared spectrometer on the Cassini spacecraft will obtain more detailed information about rotation states of Saturn's ring particles (e.g., Spilker *et al.*, 2002), further theoretical studies on rotation of ring particles and moonlets are needed for direct comparison with these observations.

In the present work, we focused on the non-gravitating case, and neglected the effect of mutual gravity between a moonlet and ring particles. Such an approximation is valid if their relative velocity is at least a few times larger than the escape velocity of the moonlet. However, in general cases, mutual gravity between ring particles plays an important role in their velocity distribution and structure formation, especially at the outer parts of planetary rings where the tidal effect becomes less important and in the case of large optical depths where collective effects become dominant (e.g., Salo, 1995; Ohtsuki, 1993, 1999; Daisaka and Ida, 1999; Ohtsuki and Emori, 2000). The effect of gravity also affects the moonlet rotation by particle collisions, as it significantly enhances the number of collisions (Morishima and Salo, 2004; Ohtsuki, 2004). On the basis of the formulation derived in the present paper, we examine the rotation of a gravitating moonlet due to particle collisions in a separate paper (Ohtsuki, 2004), where we present numerical results of three-body orbital integration that takes into account the effects of gravity and velocity distribution of ring particles. The results of the present paper may also be applied to asteroid rotation, where the effect of mutual gravity is less important owing to high relative velocities, although catastrophic fragmentation needs to be taken into account in this case.

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Appendix

In this appendix, we describe the derivation of approximate analytic expressions of the integrals in Eqs. (9), (33), and (47). Greenzweig and Lissauer (1992) derived an approximate analytic expression of the integral in Eq. (9). The idea behind their method of derivation is that the elliptic integral is a slowly varying function of its argument in the integration range, therefore it can be taken out of the integral and its argument can be replaced by an average value. In the first-order approximation, $I(\beta)$ given in Eq. (9) can be written as

$$\begin{aligned} I(\beta) &= 4\int E(A(\chi))W_1(\chi)d\chi \\ &\simeq 4E(\overline{A_1})\int W_1(\chi)d\chi, \end{aligned} \tag{A.1}$$

where

$$\begin{aligned} A(\chi) &\equiv \frac{\sqrt{3}}{2}\sqrt{1-\chi^2}, \\ W_1(\chi) &\equiv \frac{1}{\{\beta+(\beta^{-1}-\beta)\chi^2\}^2}, \end{aligned} \tag{A.2}$$

and

$$\overline{A_1} \equiv \frac{\int A(\chi)W_1(\chi)d\chi}{\int W_1(\chi)d\chi}. \tag{A.3}$$

The integrals in Eq. (A.3) can be evaluated analytically. When $\beta < 1$, we have

$$\begin{aligned} \int A(\chi)W_1(\chi)d\chi &= \frac{\sqrt{3}\pi}{8\beta}, \\ \mathcal{I}_{w1} \equiv \int W_1(\chi)d\chi &= \frac{1}{2}(1+T(\beta)), \end{aligned} \tag{A.4}$$

where

$$T(\beta) \equiv \frac{\tan^{-1}\sqrt{\beta^{-2}-1}}{\beta\sqrt{1-\beta^2}}. \tag{A.5}$$

Thus, we have (Greenzweig and Lissauer, 1992)

$$\overline{A_1} = \frac{\sqrt{3}\pi}{4\beta}(1+T(\beta))^{-1}. \tag{A.6}$$

When $\beta = 1$, we have $\overline{A_1} = \sqrt{3}\pi/8$ and $\mathcal{I}_{w1} = 1$. Greenzweig and Lissauer (1992) also calculated the second-order approximation for $I(\beta)$. Here, we only show the calculations for the first-order approximation, because they provide sufficiently accurate results, as shown below. From Eqs. (A.1), (A.4), and (A.6), we obtain (Greenzweig and Lissauer, 1992)

$$I(\beta) \simeq 4E(\overline{A_1})\mathcal{I}_{w1}. \tag{A.7}$$

The asymptotic form of $I(\beta)$ in the limit of $\beta \ll 1$ can be easily obtained from Eqs. (A.4), (A.5), and (A.6) as

$$I(\beta) \simeq \pi E(\sqrt{3}/2)/\beta. \tag{A.8}$$

Approximate analytic expressions for other integrals in Eqs. (33), (38), and (47) can be also obtained in similar manners. Those for $I_{tz}(\beta)$, $I_{x^2+y^2}(\beta)$, $I_{y^2+z^2}(\beta)$, and $I_{z^2+x^2}(\beta)$ are

$$\begin{aligned} I_{tz}(\beta) &\simeq \left(2E(\overline{A_1}) - \frac{1}{2}K(\overline{A_1}) + \frac{1}{6\pi}\right)\mathcal{I}_{w1} - K(\overline{A_1})\mathcal{I}_{2w1}, \\ I_{x^2+y^2}(\beta) &\simeq 3E(\overline{A_1})\mathcal{I}_{w1} - K(\overline{A_1})\mathcal{I}_{2w1}, \\ I_{y^2+z^2}(\beta) &\simeq \left(\frac{5}{3}E(\overline{A_1}) + \frac{1}{3}K(\overline{A_1})\right)\mathcal{I}_{w1} + K(\overline{A_1})\mathcal{I}_{2w1}, \\ I_{z^2+x^2}(\beta) &\simeq \left(\frac{10}{3}E(\overline{A_1}) - \frac{1}{3}K(\overline{A_1})\right)\mathcal{I}_{w1}, \end{aligned} \tag{A.9}$$

where

$$\mathcal{I}_{2w1} \equiv \int \chi^2 W_1(\chi)d\chi = \begin{cases} \dfrac{T(\beta)-1}{2(\beta^{-2}-1)} & \text{for } \beta < 1 \\ \dfrac{1}{3} & \text{for } \beta = 1 \end{cases} \tag{A.10}$$

In order to derive the approximate exprresions for $I_2(\beta)$ and $I_{z2}(\beta)$, we introduce

$$W_2(\chi) \equiv \frac{\beta}{\{\beta + (\beta^{-1} - \beta)\chi^2\}^3}, \quad \text{(A.11)}$$

and define $\mathcal{I}_{w2}$, $\mathcal{I}_{2w2}$, and $\overline{A_2}$, replacing W_1 with W_2 in the definitions of $\mathcal{I}_{w1}$, $\mathcal{I}_{2w1}$, and $\overline{A_1}$. $\mathcal{I}_{w2}$, $\mathcal{I}_{2w2}$, and $\overline{A_2}$ for $\beta < 1$ are evaluated as

$$\begin{aligned} \mathcal{I}_{w2} &= \frac{1}{8}(3 + 2\beta^2 + 3T(\beta)), \\ \mathcal{I}_{2w2} &= \frac{\beta^2}{8(1-\beta^2)}(1 - 2\beta^2 + T(\beta)), \\ \overline{A_2} &= \frac{\sqrt{3}\pi(3+\beta^2)}{4\beta(3 + 2\beta^2 + 3T(\beta))}. \end{aligned} \quad \text{(A.12)}$$

For $\beta = 1$, we have $\mathcal{I}_{w2} = 1$, $\mathcal{I}_{2w2} = 1/3$, and $\overline{A_2} = \sqrt{3}\pi/8$. Using these, we obtain

$$\begin{aligned} I_2(\beta) &\simeq \left(4E(\overline{A_2}) - 3F(\overline{A_2})\right)\mathcal{I}_{w2} + 3F(\overline{A_2})\mathcal{I}_{2w2} \\ I_{z2}(\beta) &\simeq \left(2E(\overline{A_2}) - \frac{3}{2}F(\overline{A_2})\right)(\mathcal{I}_{w2} - \mathcal{I}_{2w2}). \end{aligned} \quad \text{(A.13)}$$

Note that $\overline{A_1}$ and $\overline{A_2}$ as well as their elliptic integrals such as $E(\overline{A_1})$ and $E(\overline{A_2})$ agree with each other within 4% for $\beta \leq 1$, thus $\overline{A_1}$, which has a simpler expression, may be used in Eqs. (A.13) instead of $\overline{A_2}$.

In Fig. 1, we plot the values of the integrals calculated by these approximate analytic expressions (Eqs. (A.9) and (A.13)) by the open circles. We confirmed that they agree with those obtained by direct numerical integration within 5% for the entire range of β shown in Fig. 1.

In the limit of $\beta \ll 1$, the above analytic expressions have the following asymptotic forms:

$$\begin{aligned} I_{tz}(\beta) &\simeq \left(2E(\sqrt{3}/2) - \frac{1}{2}K(\sqrt{3}/2) + \frac{1}{6\pi}\right)\frac{\pi}{4\beta}, \\ I_{x^2+y^2}(\beta) &\simeq 3E(\sqrt{3}/2)\frac{\pi}{4\beta}, \\ I_{y^2+z^2}(\beta) &\simeq \left(5E(\sqrt{3}/2) + K(\sqrt{3}/2)\right)\frac{\pi}{12\beta}, \\ I_{z^2+x^2}(\beta) &\simeq \left(10E(\sqrt{3}/2) - K(\sqrt{3}/2)\right)\frac{\pi}{12\beta}, \\ I_2(\beta) \simeq 2I_{z2}(\beta) &\simeq \left(E(\sqrt{3}/2) - \frac{3}{4}F(\sqrt{3}/2)\right)\frac{3\pi}{4\beta}. \end{aligned} \quad \text{(A.14)}$$

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