On the validity of Akasofu's ε parameter and of the Vasyliunas *et al.* general formula for the rate of solar wind-magnetosphere energy input

Takao Aoki

Nagano Technical High School, Nagano 380-0948, Japan

The ε parameter of Akasofu and the general formula of Vasyliunas *et al.* (1982) have been widely used as functions representing the rate of the solar wind energy input into the Earth's magnetosphere. These two functions have a feature that dependence on the interplanetary magnetic field (IMF) is assumed to be expressed by the combination of B, the magnitude of the IMF, and $\sin(\theta/2)$, where θ is the clock angle, or by the combination of $B_T = (By^2 + Bz^2)^{1/2}$, the magnetic field perpendicular to the Sun-Earth line, and $\sin(\theta/2)$, not by the Bz component. We check the validity of this assumption by presuming that the AL index has good correlation with the solar wind energy input rate and by applying the regression analysis in a power law form to hourly values of the AL index and of solar wind parameters during the period 1966–1987. We show that Bz generally gives high correlation coefficients and shows higher consistency in exponents than the combination of B (or B_T) and $\sin(\theta/2)$. Thus we conclude that the IMF dependence of the rate of the solar wind energy input is described better by Bz than by the combination of B (or B_T) and $\sin(\theta/2)$.
Key words: Substorm, coupling function, ε parameter, Vasyliunas *et al.* general formula, IMF clock angle.

1. Introduction

The magnetospheric substorm is a basic mode of the interaction between the solar wind and the Earth's magnetosphere. The function representing the rate of the solar wind energy input into the magnetosphere is called the energy coupling function between the solar wind and the magnetosphere, and its functional form has been investigated from both analytical and theoretical points of view over four decades (e.g., Baker, 1986; Koskinen and Tanskanen, 2002).

Simple functions proposed so far are BzV (Rostoker *et al.*, 1972), where Bz is the north-south component of the interplanetary magnetic field (IMF), and V is the solar wind velocity, and BsV^2 (Murayama and Hakamada, 1975), where Bs is the southward component of the IMF: $Bs = -Bz$ for $Bz < 0$, and $Bs = 0$ for $Bz \geq 0$. The above two expressions have a feature that they are linearly dependent on Bz.

The ε parameter of Akasofu is also a widely used function (Perreault and Akasofu, 1978; Akasofu, 1979). It is expressed by $\varepsilon = VB^2 \sin^4(\theta/2)l_0^2$, where l_0 is 7 R_E, B is the magnitude of the IMF, and θ is the IMF clock angle defined as

$$\theta = \tan^{-1}(|By/Bz|) \qquad \text{for} \quad Bz > 0$$
$$\theta = 180° - \tan^{-1}(|By/Bz|) \qquad \text{for} \quad Bz < 0.$$

Thus the IMF dependence of ε is expressed by $B^2 \sin^4(\theta/2)$. Another well-known function is the general

formula of Vasyliunas *et al.* (1982) derived from dimensional analysis on the magnetohydrodynamic (MHD) flow. The IMF dependence of the Vasyliunas *et al.* equation is expressed by a function of $M_A \cdot f(\theta)$, where M_A is the Alfven Mach number, $M_A = VD^{1/2}/B_T$, D is the solar wind density, B_T is the magnetic field perpendicular to the Sun-Earth line, $B_T = (Bx^2 + By^2)^{1/2}$, $f(\theta)$ is a function of θ, and $f(\theta)$ is usually expressed by a function of $\sin(\theta/2)$. Hence the above two coupling functions have a feature that the IMF dependence is expressed by the combination of B (or B_T) and $\sin(\theta/2)$.

The purpose of the present paper is to check the validity of ε and of the Vasyliunas *et al.* formula by comparing dependence on Bz and dependence on B (or B_T) and $\sin(\theta/2)$ using hourly values of solar wind parameters and of the AL index, which is a measure of the intensity of substorms and also considered to be a measure of the solar wind energy input rate. We perform the regression analysis in a power law form and show that Bz is better than the combination of B (or B_T) and $\sin(\theta/2)$ in terms of correlation coefficients and of consistency in exponents.

2. Data and Procedure of Analysis
2.1 Data

We use hourly values of the AL index for the period from 1966 through 1987, except for 1976 and 1977 due to lack of the index. The hourly values of solar wind parameters are used for the same period. Thirty-minute delay of the AL index with respect to the solar wind parameters is introduced by using averages of two consecutive AL values. Hourly intervals for which all of the IMF, the solar wind velocity, the density, and the AL index are available are used for the present analysis. In the next section we present the
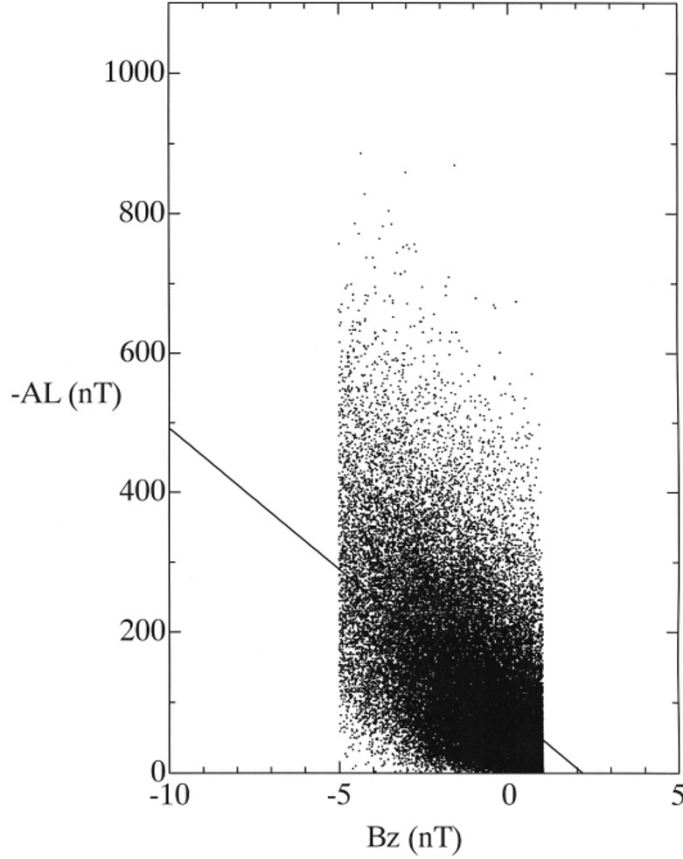
Fig. 1. Scatter plot of AL versus Bz for the data of $-5 \leq Bz < 1$ nT and $V < 600$ km/s. The solid line gives the regression line between AL and Bz.

results for the data of the intervals where $-5 \leq Bz < 1$ nT and $V < 600$ km/s. This condition for solar wind parameters is imposed to avoid extreme situations of the solar wind. Wider range of negative Bz than that of positive Bz is chosen to avoid small values of $|AL|$, because the physical meaning of them is unclear (Allen and Kroehl, 1975; Kamide and Akasofu, 1983). The IMF components are expressed in the geocentric solar magnetospheric (GSM) coordinate system.

2.2 Procedure of analysis

Figure 1 shows a scatter plot of AL versus Bz for the data of $-5 \leq Bz < 1$ nT and $V < 600$ km/s. As can be seen in this figure, the AL index develops to large negative values as Bz becomes negative. As Bz approaches zero from the negative side, AL becomes small negative values. This tendency does not cease at $Bz = 0$ and continues to some positive values of Bz. To take account of this behavior, we performed the linear regression analysis of AL on Bz, and obtained the regression line, $AL = 40.5Bz - 87.4$, with a correlation coefficient of 0.53. From these regression coefficients, we obtain $Bz_0 = 2.2$ nT as the value of the point at which the regression line crosses the abscissa. Below we use $Bz' = Bz_0 - Bz$ instead of Bz.

We performed the regression analysis in the form of $Y = A_0 X_1^{\alpha 1}$ for each of the following parameters: Bz', B, B_T, $\sin(\theta/2)$, V, M_A, and $M'_A = V D^{1/2}/B$, the Alfven Mach number defined in terms of B insted of B_T. We did this analysis by transforming $Y = A_0 X_1^{\alpha 1}$

into $\log(Y) = \log(A_0) + \alpha_1 \log(X_1)$ and by applying the linear regression analysis to this equation. We also performed the regression analysis in the form of $Y = A_0 X_1^{\alpha 1} X_2^{\alpha 2}$ for the following two-paramerer combinations: (Bz', V), $(B, \sin(\theta/2))$, $(B_T, \sin(\theta/2))$, $(M_A, \sin(\theta/2))$, and $(M'_A, \sin(\theta/2))$. Furthermore we did the regression analysis in the form of $Y = A_0 X_1^{\alpha 1} X_2^{\alpha 2} X_3^{\alpha 3}$ for the following three-parameter combinations: (Bz', V, D), $(B, \sin(\theta/2), V)$, and $(B_T, \sin(\theta/2), V)$. We will compare the exponents and the correlation coefficients in the next section.

3. Results

Table 1 shows the exponents and correlation coefficients obtained by the power law fit analysis for various kinds of solar wind parameters and of their combinations. First, we compare the correlation coefficients among Bz', $(B, \sin(\theta/2))$, and $(B_T, \sin(\theta/2))$. We notice that Bz' gives the best correlation coefficient among above three quantities. This feature remains valid when we include V: the combination (Bz', V) gives the highest correlation coefficient among (Bz', V), $(B, \sin(\theta/2), V)$, and $(B_T, \sin(\theta/2), V)$. Thus we can consider that Bz' is the best parameter among Bz', $(B, \sin(\theta/2))$, and $(B_T, \sin(\theta/2))$ in terms of correlation coefficients.

We also notice the following points from Table 1:

1) The highest correlation coefficient in Table 1 is ob-

Table 1. Exponents and correlation coefficients (c.c.) for various kinds of solar wind parameters and of their combinations obtained by the regression analysis. In this table, $(B_T, \sin(\theta/2), V)$, for example, represents the case of the regression equation of log $(-AL) = $ const. $+ \alpha_1 \log(B_T) + \alpha_2 \log(\sin(\theta/2)) + \alpha_3 \log(V)$. Errors are the standard errors. The data used for every case of solar wind parameters and of their combinations are the same; the number of data points is 47,740.

parameters	exponents			c.c.*
	α_1	α_2	α_3	
Bz'	1.107 ± 0.009			0.508
B	1.021 ± 0.011			0.404
B_T	0.492 ± 0.006			0.350
$\sin(\theta/2)$	1.050 ± 0.014			0.317
V	2.272 ± 0.022			0.423
M_A	-0.317 ± 0.006			0.232
M_A'	-0.417 ± 0.010			0.183
(Bz', V)	1.118 ± 0.007	2.303 ± 0.018		0.665
$(B, \sin(\theta/2))$	0.929 ± 0.010	0.884 ± 0.013		0.483
$(B_T, \sin(\theta/2))$	0.398 ± 0.006	0.783 ± 0.014		0.417
$(M_A, \sin(\theta/2))$	-0.219 ± 0.006	0.912 ± 0.015		0.353
$(M_A', \sin(\theta/2))$	-0.351 ± 0.010	1.002 ± 0.014		0.352
(Bz', V, D)	1.088 ± 0.007	2.680 ± 0.020	0.247 ± 0.005	0.682
$(B, \sin(\theta/2), V)$	0.754 ± 0.009	0.921 ± 0.012	1.985 ± 0.020	0.604
$(B_T, \sin(\theta/2), V)$	0.400 ± 0.005	0.788 ± 0.013	2.284 ± 0.020	0.596

*Correlation coefficient.

tained for the combination of (Bz', V, D).

2) The correlation coefficients for M_A, M_A', $(M_A, \sin(\theta/2))$, and $(M_A', \sin(\theta/2))$ are very small compared with those for Bz', $(B, \sin(\theta/2))$, and $(B_T, \sin(\theta/2))$.

3) The exponent for Bz' is approximately equal to unity. This feature is seen in common in all cases of Bz', (Bz', V), and (Bz', V, D).

4) The exponent for B is roughly equal to unity in the cases of B, $(B, \sin(\theta/2))$, and $(B, \sin(\theta/2), V)$. It is never equal to two, as expected from ε.

5) The exponent for B_T is roughly equal to 0.5 in the cases of B_T, $(B_T, \sin(\theta/2))$, and $(B_T, \sin(\theta/2), V)$.

6) The exponent for $\sin(\theta/2)$ is roughly equal to unity in all cases including $\sin(\theta/2)$: $\sin(\theta/2)$, $(B, \sin(\theta/2))$, $(B_T, \sin(\theta/2))$, $(M_A, \sin(\theta/2))$, $(M_A', \sin(\theta/2))$, $(B, \sin(\theta/2), V)$, and $(B_T, \sin(\theta/2), V)$. It is never equal to four, as expected from ε.

7) The exponent for V is roughly equal to two, as in BsV^2 obtained by Murayama and Hakamada (1975), not to unity, as expected from ε. This feature is seen in common in all cases including V: V, (Bz', V), (Bz', V, D), $(B, \sin(\theta/2), V)$, and $(B_T, \sin(\theta/2), V)$.

4. Discussion
4.1 Influence of the conditions of data selection on the results

We obtained the results in the preceding section under the condition of $-5 \leq Bz < 1$ nT and $V < 600$ km/s. In this subsection we discuss the influence of the conditions of data selection on the results.

We performed the same analysis as the one in the preceding section by changing the conditions for the solar wind parameters. As for the condition on Bz we investigated the following seventeen ranges: $[-3, 1]$ (which means $-3 \leq Bz < 1$ nT; we use the same format below), $[-4, 1]$, $[-5, 1]$, $[-6, 1]$, $[\ , 1]$, $[-4, 0]$, $[-5, 0]$, $[-6, 0]$, $[-7, 0]$, $[\ , 0]$, $[-5, -1]$, $[-6, -1]$, $[-7, -1]$, $[\ , -1]$, $[-5, -2]$, $[-6, -2]$, and $[\ , -2]$. As for V, we examined the following three ranges: $V < 600$ km/s, $V \geq 600$ km/s, and all V. We analysed all cases of any combinations of the above Bz and V ranges.

Table 2 shows the examples of this analysis: results for Bz', for $\sin(\theta/2)$, and for the combination of $(B_T, \sin(\theta/2))$ for various ranges of Bz and for the range of $V < 600$ km/s. From this table we notice that in some Bz ranges the combination of $(B_T, \sin(\theta/2))$ shows slightly better correlation coefficients than Bz'. However, the exponents for B_T and for $\sin(\theta/2)$ show larger variability for different ranges of Bz than those for Bz': the exponents for Bz' range from 1.09 to 1.30, those for B_T from 0.31 to 0.91, and those for $\sin(\theta/2)$ from 0.63 to 4.40 (from 0.37 to 3.04 for single parameter case). Thus the exponents for Bz show higher consistency than those for B_T and for $\sin(\theta/2)$ in the combination of $(B_T, \sin(\theta/2))$ for the change in the range of Bz.

The above feature was more generally confirmed as follows: The combinations of (Bz', V) and of (Bz', V, D) always show good correlation coefficients, and (Bz', V, D), in paticular, almost always gives the best correlation coefficient among the solar wind parameters and their combinations listed in Table 1 for every range of Bz. The combinations of $(B_T, \sin(\theta/2), V)$ and of $(B, \sin(\theta/2), V)$ usually show good correlation coefficients, and $(B_T, \sin(\theta/2), V)$,

Table 2. Exponents and correlation coefficients (c.c.) for Bz', for $\sin(\theta/2)$, and for the combination of B_T and $\sin(\theta/2)$ for various ranges of Bz and for $V < 600$ km/s.

Bz range	Number of data points	Bz_0	Bz'		$\sin(\theta/2)$		$(B_T, \sin(\theta/2))$		
			α	c.c.*	α	c.c.*	α_1	α_2	c.c.*
$[-3, \quad 1]$	40818	2.5	1.13 ± 0.01	0.405	0.80 ± 0.02	0.257	0.31 ± 0.01	0.63 ± 0.02	0.340
$[-4, \quad 1]$	45056	2.3	1.11 ± 0.01	0.467	0.95 ± 0.02	0.293	0.36 ± 0.01	0.73 ± 0.02	0.384
$[-5, \quad 1]$	47740	2.2	1.11 ± 0.01	0.508	1.05 ± 0.01	0.317	0.40 ± 0.01	0.78 ± 0.01	0.417
$[-6, \quad 1]$	49407	2.1	1.09 ± 0.01	0.535	1.12 ± 0.01	0.332	0.43 ± 0.01	0.82 ± 0.01	0.441
$[\quad , \quad 1]$	52270	2.3	1.17 ± 0.01	0.596	1.28 ± 0.01	0.362	0.50 ± 0.01	0.85 ± 0.01	0.497
$[-4, \quad 0]$	32971	1.8	1.20 ± 0.01	0.423	1.74 ± 0.05	0.196	0.58 ± 0.01	2.96 ± 0.05	0.438
$[-5, \quad 0]$	35655	1.7	1.18 ± 0.01	0.473	2.15 ± 0.05	0.243	0.61 ± 0.01	3.13 ± 0.04	0.483
$[-6, \quad 0]$	37322	1.7	1.18 ± 0.01	0.505	2.42 ± 0.04	0.273	0.62 ± 0.01	3.22 ± 0.04	0.512
$[-7, \quad 0]$	38330	1.7	1.19 ± 0.01	0.528	2.61 ± 0.04	0.293	0.63 ± 0.01	3.28 ± 0.04	0.533
$[\quad , \quad 0]$	40185	2.1	1.27 ± 0.01	0.575	3.04 ± 0.04	0.336	0.65 ± 0.01	3.37 ± 0.04	0.580
$[-5, -1]$	23383	1.4	1.18 ± 0.02	0.398	0.93 ± 0.07	0.092	0.82 ± 0.01	3.88 ± 0.08	0.412
$[-6, -1]$	25050	1.5	1.20 ± 0.02	0.440	1.29 ± 0.06	0.126	0.82 ± 0.01	3.90 ± 0.07	0.452
$[-7, -1]$	26058	1.5	1.20 ± 0.01	0.470	1.54 ± 0.06	0.149	0.83 ± 0.01	3.93 ± 0.06	0.480
$[\quad , -1]$	27913	2.3	1.30 ± 0.01	0.534	2.17 ± 0.06	0.205	0.81 ± 0.01	3.87 ± 0.06	0.545
$[-5, -2]$	13657	1.2	1.20 ± 0.03	0.314	0.37 ± 0.09	0.035	0.91 ± 0.02	4.40 ± 0.13	0.323
$[-6, -2]$	15324	1.4	1.21 ± 0.03	0.365	0.70 ± 0.09	0.065	0.89 ± 0.02	4.29 ± 0.11	0.372
$[\quad , -2]$	18187	2.8	1.30 ± 0.02	0.491	1.70 ± 0.08	0.151	0.83 ± 0.01	4.00 ± 0.08	0.500

*Correlation coefficient.

in paticular, sometimes shows nearly the same level of correlation coefficient as (Bz', V, D). However, the exponents for B_T and for $\sin(\theta/2)$ in the combination of $(B_T, \sin(\theta/2), V)$ show larger variability than those for Bz'. Also the exponents for B and for $\sin(\theta/2)$ in the cases of B, $(B, \sin(\theta/2))$, and $(B, \sin(\theta/2), V)$ show larger variability than those for Bz'. Hence we have reached the same conclusion, again, as in the preceding section that Bz generally gives high correlation coefficients and shows higher consistency in exponents than the combination of B_T (or B) and $\sin(\theta/2)$ from the analysis for the various kinds of conditions on the data selection.

Here we would like to point out one fact. There exist some cases where the exponents for $\sin(\theta/2)$ are roughly four (e.g., Table 2), but even in those cases the exponents for B_T (or B) are not two, as expected from the ε parameter. Thus we never obtained the $B_T^2 \sin^4(\theta/2)$ (or $B^2 \sin^4(\theta/2)$) dependence from the present analysis.

In the above, we imposed restrictions on the basis of the values of Bz and made logarithmic analyses mainly in the negative range of Bz. This methodology itself might work in favor of Bz and produce the result of a better performance of Bz. In order to check this possibility, we imposed restrictions on the basis of the values of θ and chose the ranges so that each of the ranges is symmetric with respect to $90°$: $90 - 10p \leq \theta < 90 + 10p°$, ($p = 1, 2, \cdots, 9$). For each range of θ, we performed the linear regression analysis of AL on $V^l B^m \sin^n(\theta/2)$, ($l = 0, 1$; $m = 0, 2$; $n = 1, 2, 4$), on $V^l B_T^m \sin^n(\theta/2)$, ($l = 0, 1$; $m = 0, 2$; $n = 1, 2, 4$), and on $V^l Bzc$, ($l = 0, 2$), where Bzc is defined as

$$Bzc = 2.2 - Bz \quad \text{for} \quad Bz < 2.2 \text{ nT}$$
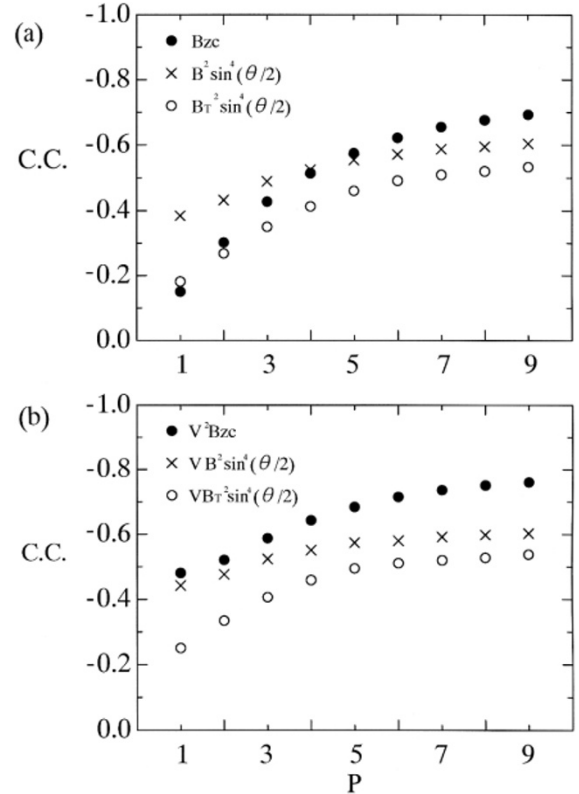$$Bzc = 0 \quad \text{for} \quad Bz \geq 2.2 \text{ nT}.$$

Fig. 2. (a) Correlation coefficients (c.c.) of AL with Bzc (solid circles), with $B^2 \sin^4(\theta/2)$ (crosses), and with $B_T^2 \sin^4(\theta/2)$ (open circles) for the ranges of $90 - 10p \leq \theta < 90 + 10p°$, ($p = 1, 2, \cdots, 9$) and for the range of $V < 600$ km/s. Values of the abscissa are the values of p. (b) Correlation coefficients (c.c.) of AL with $V^2 Bzc$ (solid circles), with $V B^2 \sin^4(\theta/2)$ (crosses), and with $V B_T^2 \sin^4(\theta/2)$ (open circles) for the ranges of $90 - 10p \leq \theta < 90 + 10p°$, ($p = 1, 2, \cdots, 9$) and for the range of $V < 600$ km/s.

Table 3. Linear correlation coefficients (c.c.) between solar wind parameters for various ranges of Bz and for $V < 600$ km/s. In this table, $\langle Bz, \sin(\theta/2)\rangle$, for example, represents the case for the linear regression analysis between Bz and $\sin(\theta/2)$. The same data as those in Table 2 are used for each range of Bz.

Bz range	Number of data points	$\langle Bz, \sin(\theta/2)\rangle$	$\langle Bz, B\rangle$	$\langle Bz, B_T\rangle$	$\langle Bz, V\rangle$	$\langle \sin(\theta/2), B\rangle$	$\langle \sin(\theta/2), B_T\rangle$	$\langle \sin(\theta/2), V\rangle$	$\langle B, B_T\rangle$	$\langle B, V\rangle$	$\langle B_T, V\rangle$
$[-3,\ 1]$	40818	-0.768	-0.136	-0.190	0.015	0.010	0.027	-0.005	0.763	0.162	-0.007
$[-4,\ 1]$	45056	-0.758	-0.212	-0.282	0.014	0.039	0.065	-0.005	0.775	0.156	-0.008
$[-5,\ 1]$	47740	-0.746	-0.274	-0.359	0.012	0.068	0.103	-0.005	0.785	0.154	-0.005
$[-6,\ 1]$	49407	-0.734	-0.328	-0.419	0.013	0.094	0.134	-0.007	0.794	0.152	-0.005
$[\ \ ,\ 1]$	52270	-0.651	-0.550	-0.630	-0.008	0.176	0.218	-0.005	0.843	0.149	0.014
$[-4,\ 0]$	32971	-0.629	-0.246	-0.324	0.011	-0.248	-0.370	0.006	0.777	0.152	-0.008
$[-5,\ 0]$	35655	-0.637	-0.308	-0.402	0.008	-0.200	-0.294	0.004	0.789	0.151	-0.005
$[-6,\ 0]$	37322	-0.638	-0.364	-0.464	0.009	-0.155	-0.233	0.002	0.799	0.149	-0.004
$[-7,\ 0]$	38330	-0.636	-0.410	-0.511	0.009	-0.121	-0.187	0.001	0.810	0.147	-0.002
$[\ \ ,\ 0]$	40185	-0.583	-0.599	-0.680	-0.017	-0.003	-0.044	0.003	0.853	0.149	0.020
$[-5, -1]$	23383	-0.410	-0.295	-0.389	0.001	-0.417	-0.576	0.019	0.789	0.147	-0.001
$[-6, -1]$	25050	-0.432	-0.358	-0.461	0.003	-0.367	-0.502	0.015	0.800	0.147	0.001
$[-7, -1]$	26058	-0.443	-0.410	-0.516	0.002	-0.326	-0.442	0.013	0.812	0.145	0.004
$[\ \ , -1]$	27913	-0.429	-0.629	-0.711	-0.031	-0.163	-0.231	0.014	0.860	0.152	0.033
$[-5, -2]$	13657	-0.252	-0.256	-0.342	-0.005	-0.558	-0.738	0.020	0.794	0.149	0.005
$[-6, -2]$	15324	-0.288	-0.327	-0.426	-0.001	-0.510	-0.664	0.014	0.803	0.151	0.008
$[\ \ , -2]$	18187	-0.327	-0.652	-0.733	-0.048	-0.263	-0.324	0.009	0.868	0.165	0.052

We did this analysis for the following three ranges of V: $V < 600$ km/s, $V \geq 600$ km/s, and all V.

Figures 2(a) and 2(b) show the correlation coefficients between AL and various parameters for different ranges of θ and for $V < 600$ km/s. Both figures clearly show the general superiority of Bz over $B^2 \sin^4(\theta/2)$ and over $B_T^2 \sin^4(\theta/2)$. Other results (not shown) also support the conclusion that Bz is better than the combination of B (or B_T) and $\sin(\theta/2)$.

4.2 Influence of solar wind parameters other than Bz and V on the results

In the Results section we mainly analysed the effects of Bz and of V. There exist, however, other solar wind parameters which are known to influence the value of AL. Those are the solar wind density (e.g., Maezawa, 1978), the IMF By component, and the dipole tilt angle (χ) (Aoki, 1977; Hakamada *et al.*, 1980; Murayama *et al.*, 1980). The dipole tilt angle is defined as the geomagnetic latitude of the subsolar point. In this subsection we discuss the influence of these parameters on the results.

First, we consider the effect of the solar wind density. As can be seen in (Bz', V, D) in Table 1, D really improves the correlation coefficient compared with the correlation coefficient of (Bz', V), but this improvement is very small. The exponent for the density is also very small. Thus we can expect that the density would not affect the results of the preceding section.

Second, we discuss the effects of By and of χ. The main effect of By is almost linear for a given sign of χ, and the gradient of the regression line reverses its sign for different signs of χ (Hakamada *et al.*, 1980). Below we call this phenomenon the By-χ effect. This effect also would have little influence on our results, because this would be averaged out by using means for more than one year because of the annual periodicity of the value of χ. The dipole tilt angle also has an effect of maximizing geomagnetic activity at $\chi = 0°$. This effect would also be leveled off by averaging data for more than one year. Thus we can expect that By and χ do not change the results in the preceding section.

On the other hand, the existence of the By-χ effect shows a great difficulty to the idea that the IMF dependence of the AL index can be described by the combination of B (or B_T) and $\sin(\theta/2)$. The reason is as follows: Each of the following three quantities, B, B_T, and $\sin(\theta/2)$, is symmetric with respect to the sign of By (i.e., concerning the influence of By, each quantity is determined by the absolute value of By and has no relation to the sign of By), while obviously the By-χ effect is not symmetric with respect to it. Thus the By-χ effect cannot be described by the combination of B (or B_T) and $\sin(\theta/2)$.

4.3 Influence of intercorrelations among solar wind parameters

It is well-known that solar wind parameters show intercorrelations among themselves. In this subsection we discuss the influence of these correlations on the results.

We performed the linear regression analysis for any combinations of the following solar wind paramers: Bz, $\sin(\theta/2)$, B, B_T, and V. Table 3 shows the correlation coefficients for various ranges of Bz and for $V < 600$ km/s. Correlation coefficients for the ranges of $V \geq 600$ km/s

and for all V are almost the same as those in Table 3 (not shown).

From this table we notice that the correlation coefficients for the combinations containing V are generally very small: correlation coefficients between V and B are slightly positive values, and correlation coefficients for other combinations are almost zero. This fact guarantees that we can discuss dependence on the IMF without considering the existence of dependence on V in the coupling function.

4.4 Remaining problems

In the present paper, we have used the AL index as a measure of the rate of the solar wind energy input into the magnetosphere. Actually the AL index reflects the intensity of the westward electrojets flowing in the ionosphere of the auroral zone. The energy transferred through the solar wind-magnetosphere interaction is distributed to various regions in the magnetosphere and produces a lot of phenomena other than the auroral electrojets, such as ring current intensification and plasmoid release. So to investigate the accurate functional form of the soalr wind-magnetosphere coupling we should compare the total energy with solar wind parameters. These comparisons, however, are not an easy task because of some uncertainties such as lack of accurate knowledge of decay function of the ring current and lack of the precise method of measuring energy carried by plasmoids. In this sense the present study is considered as a first step toward more appropriate investigations. We are now investigating the solar wind parameter dependence of the ring current injection rate and preparing a paper.

5. Conclusions

We have examined the IMF dependence of the AL index by comparing the influence of Bz and that of the combination of B (or B_T) and $\sin(\theta/2)$. The regression analysis in a power law form shows that Bz generally gives high correlation coefficients and shows higher consistency in exponents. Higher consistency of Bz can be seen in the fact that the exponent for Bz' shows smaller variability for the change in the range of Bz (Table 2). This fact implies that the IMF dependence of the AL index is described better by Bz than by the combination of B (or B_T) and $\sin(\theta/2)$, and that the ε parameter of Akasofu and the general formula of Vasyliunas *et al.* (1982) for the energy coupling function are less appropriate than a function of Bz.

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T. Aoki (e-mail: aoki317@ruby.ocn.ne.jp)