Effect of stochastic ionospheric delay modeling for GPS ambiguity resolution

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(Received July 21, 2009; Revised September 2, 2009; Accepted September 6, 2009; Online published September 30, 2009)

The network-derived ionospheric delay can improve the fast and accurate determination of the long baseline in both the rapid-static and kinematic Global Positioning System (GPS) positioning mode. In this study, an interpolation of the undifferenced (UD) ionospheric delays is performed on a satellite-by-satellite and epoch-by-epoch basis, respectively, using the least-squares collocation (LSC) to provide not only ionospheric delays but also their variances. The developed method retains the simplicity of the two-dimensional (2-D) model, but it does not introduce errors due to the thin-shell assumption made in the single-layered model. Our method also provides flexibility in forming the predicted double-differenced (DD) ionospheric delays. Faster and more reliable positioning solutions can be obtained when the developed method is used to predict DD ionospheric delays. The numerical test applying the method to the Ohio Continuously Operating Reference Station network shows a 23% improvement in mean time-to-fix with the network-derived ionospheric delays.

Key words: GPS positioning, ionosphere modeling, least-squares collocation, CORS.

1. Introduction

Over the past two decades, the Global Positioning System (GPS) has been widely used for analyses in the fields of solid Earth science, atmospheric science, ocean science, and surface deformation. Some GPS applications require fast and precise positioning due to the limited time available or unfavorable conditions in specific areas (e.g., intertidal zones). Positioning using GPS is usually performed in double-differenced (DD) mode, with the quality of the GPS precise positioning being highly dependent on the successful resolution of the carrier phase ambiguities. One of the problems encountered when attempting to resolve the ambiguity is the high correlation between the ionospheric delay and the ambiguity in the GPS measurements. Consequently, measurements over a considerable time span are required to resolve the ambiguities (Odijk, 2000). To obtain a converging solution within a short time span, researchers have investigated and analyzed the use of external ionospheric information derived from a permanent GPS network, such as the Ohio Continuously Operating Reference Station (CORS) network (Odijk *et al.*, 2000; Cannon *et al.*, 2001; Yi and Grejner-Brzezinska, 2003; Grejner-Brzezinska *et al.*, 2005). In modeling the ionospheric effects, numerous ionospheric maps from the reference GPS network, which are based mainly on either single-layered or tomographic models, have been developed for user positioning (Schaer, 1999; Wielgosz *et al.*, 2003; Spencer *et al.*, 2004). However, these models are limited in terms of providing stochastic information, i.e., they provide only the ionospheric delay from the model, with no corresponding error estimation. As such, positioning with the constraint of ionospheric delay may cause a biased solution or incorrect ambiguity resolution (AR) in DD mode (Grejner-Brzezinska *et al.*, 2004). To overcome this limitation, Kriging or least-squares collocation (LSC), which provides not only the interpolated value for the ionospheric delay but also its variance, has been proposed as an alternative. Orus *et al.* (2005) and Blanch *et al.* (2004) applied the Kriging interpolation to a single- and multi-layered ionospheric model, respectively. The Kriging or LSC has also been successfully applied to the interpolation of the DD ionospheric delays (Odijk *et al.*, 2000; Yi and Grejner-Brzezinska, 2003). However, to date, there has been no attempt to use stochastic information on the interpolated value for the positioning.

Here, the prediction of undifferenced (UD) ionospheric delays together with error variances through the LSC is reported on a satellite-by-satellite and an epoch-by-epoch basis, respectively. In this way, the simplicity of the two-dimensional (2-D) model is still retained, but no error sources due to a thin-shell assumption in single-layered model are introduced. Furthermore, the prediction of UD ionospheric delay provides flexibility in forming the predicted DD ionospheric delays, i.e., in the selection of a satellite pair for the generation of DD ionospheric delays. In addition to these advantages, the developed method can be efficiently utilized in user positioning. In other words, we found that the integer ambiguity would be obtained in shorter time span with the provision of the predicted ionospheric delay with its error variance. The improvement in ambiguity resolution using the proposed algorithm is demonstrated using the data collected from the Ohio CORS network.

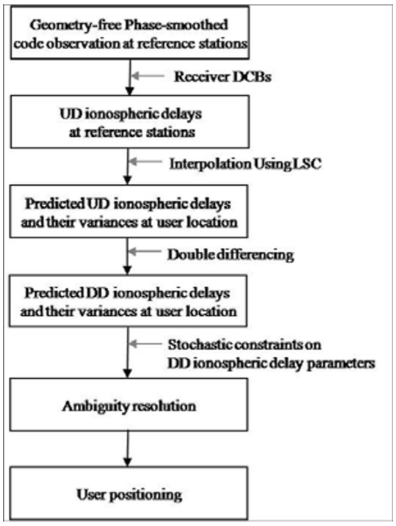

Fig. 1. Overall procedures adopted to demonstrate and analyze the proposed method.

2. Methodology

In order to perform network-based ionospheric delay modeling, it is necessary to extract UD ionospheric delay information from the GPS measurements. Consequently, UD ionospheric delays at each reference station are extracted from the GPS measurement in the form of geometry-free phase-smoothed P-code observations. The receiver differential code bias (DCB) is estimated and removed to obtain UD ionospheric delays using the method proposed by Hong *et al.* (2008). Interpolation of the UD ionospheric delays is then performed using LSC so that the predicted UD ionospheric delays together with their variances are obtained at the user location. The DD ionospheric delays and their variances at the user location are computed by a simple differencing operation and the law of error propagation, respectively. This information is used for user positioning, and the performance of user positioning is analyzed in terms of the ambiguity time-to-fix. Figure 1 shows the overall procedures adopted in this study.

The geometry-free phase-smoothed P-code observations are used to estimate UD ionospheric delays at the reference stations, and the corresponding formula is shown in Eq. (1) (Hong *et al.*, 2008).

$$E\left\{\tilde{P}_i^k\right\} = E\left\{\tilde{P}_{i,1}^k - \tilde{P}_{i,2}^k\right\} = F \cdot I_i^k + \Delta b^k + \Delta b_i \qquad F := \left(1 - f_1^2/f_2^2\right) \tag{1}$$

where $E\{\cdot\}$ is the expectation operator; subscript i and superscript k indicate the indices of the receiver and the satellite, respectively; $\tilde{P}_i^k$ is the geometry-free phase-smoothed P-code observations; $\tilde{P}_{i,1}^k$ and $\tilde{P}_{i,2}^k$ are phase-smoothed P-code observations on L1 and L2, respectively; I_i^k is the UD ionospheric delay related to L1; Δb^k and Δb_i are satellite and receiver differential code bias (DCB), respectively; f_1 and f_2 are carrier frequencies of L1 and L2, respectively.

As shown in Eq. (1), the biased ionospheric delay ($I_i^k + \Delta b^k/F$) due to the satellite DCB from all of the network stations for one satellite can be computed at a specific epoch once the receiver DCBs are fixed to the known values. For the determination of receiver DCBs, refer to Hong *et al.* (2008). Information on the satellite DCB, Δb^k, is available from the International GNSS Service (IGS), so that the bias-free UD ionospheric delays can be obtained if needed.

Once UD ionospheric delays are obtained, the UD ionospheric delays from all of the network stations for one satellite are interpolated to the user location using LSC on an epoch-by-epoch basis. The LSC is a useful method for the interpolation or prediction of the signals as it introduces the covariance function related to the correlation of observed signals. The first-order Gauss-Markov covariance model is used in this study, and the coefficients of this covariance function are estimated from the observed UD ionospheric delays. It should be noted that the estimated coefficients of the covariance function may be different for different satellites and epochs because the covariance function is determined empirically based on the observed data. Each 10-min data span for each satellite is used for the estimation of the coefficients of the covariance function in order to retain the reliability of the estimation process. For example, any possible gap in the data at any reference station may degrade the quality of the estimated covariance function. It is also assumed that the covariance function does not change significantly within 10 min. For more details on the mathematical model and its solution for the LSC method, see Moritz (2001) and Serpas (2003).

The predicted DD ionospheric delays and their variances at the user location are easily obtained using the DD operator and the law of error propagation. Consequently, the information can be used as stochastic constraints on the DD ionospheric delay parameters in user positioning to improve the integer AR. The L1 and wide-lane linear combinations of GPS observations processed with external ionospheric delays and the least-squares ambiguity decorrelation adjustment (LAMBDA) method are applied to fix the ambiguities to their integer values (Teunissen, 1994). Also, the AR validation procedure is performed using the W-ratio test proposed by Wang *et al.* (1998).

3. Numerical Test and Results

To demonstrate the performance of the proposed method, we selected 15 geometrically well-distributed CORS stations with a separation of approximately 80 km between stations (see Fig. 2). The GPS data from 24-h datasets were

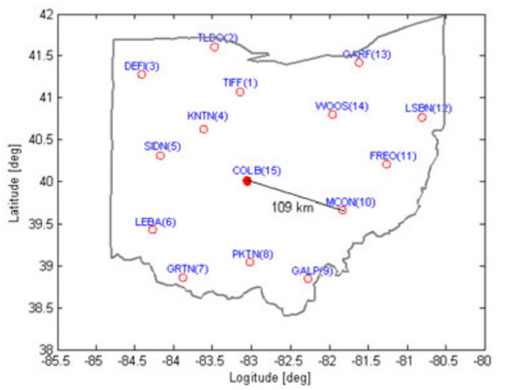

Fig. 2. Location map of the stations of the Ohio CORS network.

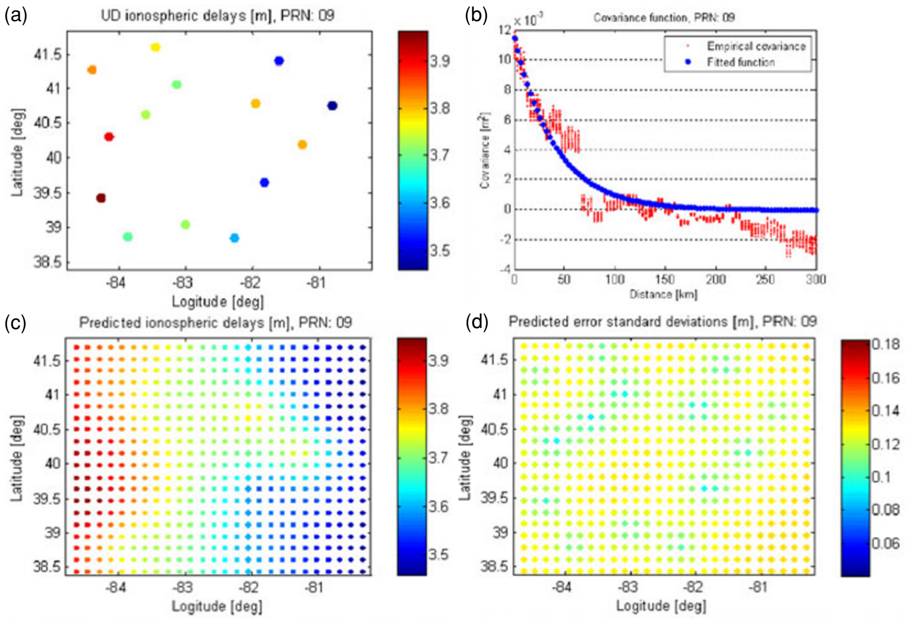

Fig. 3. (a) UD ionospheric delays (possibly biased by satellite DCBs); (b) estimated covariance function; (c) predicted UD ionospheric delays; (d) predicted error standard deviations over the Ohio CORS network for selected satellites at the first epoch (04/03/2004).

collected at a 30-s sampling rate on April 3, 2004 and processed. It should be noted that the COLB station is assumed to be the simulated user station and that all other stations are reference stations, i.e., all of the stations except for COLB are used to generate the predicted UD ionospheric delays at the location of COLB station.

As described in Section 2.1, the UD ionospheric delays at the reference stations can be obtained once receiver DCBs are properly determined, as shown in Eq. (1). The receiver DCBs for the Ohio CORS network are successfully estimated by the method of Hong *et al.* (2008) and the information used to extract UD ionospheric delays from the GPS observations. Figure 3(a) presents an example of UD ionospheric delays for PRN09 satellites during the first epoch over the network, and each dot corresponds to one network station. To show the variations in the predicted ionospheric delays over the entire Ohio area, we perform the interpolation at grid points using the empirically determined covariance function, as shown in Fig. 3(b); the consequent interpolation results are presented in Fig. 3(c). The grid interval is set to 0.17° in longitude and latitude and is selected for the efficient presentation of predicted UD ionospheric delays over the Ohio area. However, it should be pointed out that in real applications the interpolation will be performed only at the user location. The corresponding predicted error standard deviation maps are shown in Fig. 3(c). As a consequence, the predicted UD ionospheric delays and their errors are obtained at each epoch.

The impact of network-derived ionospheric delays on the fast AR are analyzed and validated by forming the predicted DD ionospheric delays for each pair of satellite and generating their variances through the law of error propagation for the baseline, MCON-COLB. This information is then used as stochastic constraints on the DD ionospheric delay parameters for the user positioning in rapid-static mode. It is notable that the baseline length between the MCON and the COLB stations is approximately 109 km. Thus, the tropospheric effects should be properly modeled so that errors due to mis-modeled tropospheric delays can be reduced. The tropospheric delay information provided by NOAA is used in this study (available at: http://gpsmet.noaa.gov/jsp/background/background.jsp).

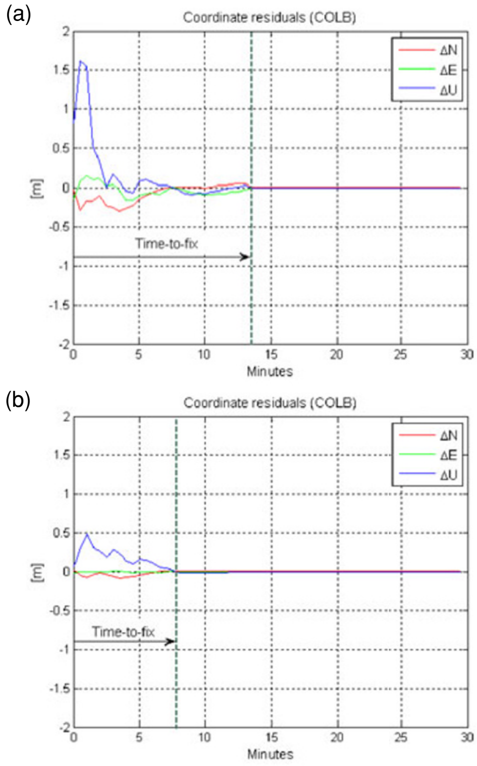

Fig. 4. Example of convergence rate of ambiguity fixed rapid-static solutions without (a) and with (b) external ionospheric delays.

The ambiguities are fixed to their integer values when

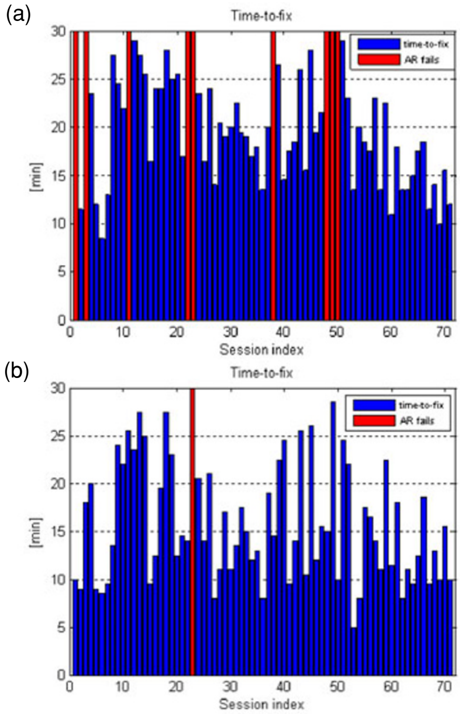

Fig. 5. Plot of time-to-fix for all sessions of rapid-static solutions (MCON-COLB baseline, 04/03/2004) without (a) and with (b) the external ionospheric delays.

the W-ratio passes the empirically determined critical value of 4, and the elapsed time is called time-to-fix. Figure 4 shows one example of an ambiguity fixed rapid-static solution without and with external ionospheric delays. The differences between the published and the estimated coordinates (coordinate residuals) are expressed by the north, east, and up components. Note that the use of external ionospheric delays for positioning significantly increases the convergence rate of the solution.

The 71 30-min sessions within the analyzed 24-h period were also processed, i.e., the estimation procedure started every 20 min, and the time-to-fix for each session was computed for both cases. As seen in Fig. 5, the time-to-fix is considerably improved in most cases when the external ionospheric delays are used in positioning. However, there were some sessions that never exceeded the predefined critical value. This may happen when the satellite geometry is changed within the session window; for example, when a new satellite is observed during the session. This means that a new DD ambiguity parameter for the new satellite pair is added and, consequently, the corresponding W-ratio value decreases. AR failed in nine of the 71 sessions when no external ionospheric delays were used, but it failed in only one session when the external ionospheric delays were used. This result clearly shows that the ionospheric delays help in the ambiguity-fixing procedure. Overall, a 23% improvement in the mean time-to-fix is obtained when external ionospheric delays are used for positioning.

4. Summary and Conclusions

An algorithm that can be used to interpolate the UD ionospheric delays using LSC is presented. The application of this method for determining user location is also demonstrated. The UD ionospheric delays with their error variances obtained from the Ohio CORS network were used to predict UD ionospheric delays at the user location. The predicted DD ionospheric delays were then formed from the predicted UD ionospheric delays for user positioning. The results showed a 23% improvement in mean time-to-fix in AR when network-derived ionospheric delays were introduced. It is therefore expected that the proposed method will significantly improve the precision and reliability of GPS positioning when only short spans of data are available.

References

Blanch, J., T. Walter, and P. Enge, Ionospheric estimation using extended Kriging for low latitude SBAS, *ION GNSS 17th International Technical Meeting of the Satellite Division*, Long Beach, CA, pp. 387–391, 21–24 Sept., 2004.

Cannon, M. E., G. Lachapelle, P. Alves, L. P. Fortes, and B. Townsend, GPS RTK positioning using a regional reference network, theory and results, *Proceedings of the 5th GNSS International Symposium*, Seville, 8–11 May, 2001.

Grejner-Brzezinska, D. A., P. Wielgosz, I. Kashani, D. A. Smith, P. S. J. Spencer, D. S. Robertson, and G. L. Mader, An analysis of the effects of different network-based ionosphere estimation models on rover positioning accuracy, *J. Global Position. Systems*, **3**(1–2), 115–131, 2004.

Grejner-Brzezinska, D. A., P. Wielgosz, and I. Kashani, Performance assessment of the new rapid-static module of the online positioning user service-OPUS-RS, *Proceedings of the 18th ION GNSS International Technical Meeting of the Satellite Division*, Long Beach, CA, pp. 2595–2605, 2005.

Hong, C.-K., D. A. Grejner-Brzezinska, and J. H. Kwon, Efficient GPS receiver DCB estimation for ionosphere modeling using satellite-receiver geometry changes, *Earth Planets Space*, **60**(11), e25–e28, 2008.

Moritz, H., *Advanced Physical Geodesy* (reprint), Dept. of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, Columbus, OH, USA, 2001.

Odijk, D., Weighting ionospheric corrections to improve fast GPS positioning over medium distances, *Proceedings of ION GPS*, Sept. 19–12, Salt Lake City, UT, pp. 1113–1123, 2000.

Odijk, D., H. Marel, and I. Song, Precise GPS positioning by applying ionospheric corrections from an active control network, *GPS Solutions*, **3**, 49–57, 2000.

Orus, R., M. Hernandez-Pajares, J. M. Juan, and J. Sanz, Improvement of global ionospheric VTEC maps by using the kriging interpolation technique, *J. Atmos. Sol.-Terr. Phys.*, **67**, 1598–1609, 2005.

Serpas, J. G., Local and regional geoid determination from vector airborne gravimetry, *OSU Report No. 468*, Department of Civil and Environmental Engineering and Geodetic Science, The Ohio State University, USA, 2003.

Shaer, S., Mapping and predicting the Earth's ionosphere using global positioning system, PhD dissertation, Astronomical Institute, University of berne, Switzerland, 205 pp., 1999.

Spencer, P. S. J., D. S. Robertson, and G. L. Mader, Ionospheric data assimilation methods for geodetic applications, *Proceedings of the IEEE PLANS*, Monterey, California, pp. 510–517, 2004.

Teunissen, P. J. G., A new method for fast carrier phase ambiguity estimation, *Proceedings of the IEEE PLANS*, pp. 562–573, 1994.

Wang, J., M. Stewart, and M. Tsakiri, A discrimination test procedure for ambiguity resolution on-the-fly, *J. Geod.*, **72**, 644–653, 1998.

Wielgosz, P., D. Grejner-Brzezinska, I. Kashani, and Y. Yi, Instantaneous regional ionosphere modeling, *Proceedings of ION GPS/GNSS 2003*, pp. 1750–1757, 2003.

Yi, Y. and D. A. Grejner-Brzezinska, Kinematic carrier phase GPS positioning aided by an instantaneous local ionospheric model based on multiple base stations using Kriging, *Proceedings of the 59th ION Annual Meeting/22nd CIGTF Guidance Test Sypositium*, Albuquerque, NM, pp. 387–396, 2003.

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