

Dominant periods of the 2004 Sumatra tsunami and the estimated source size

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Dominant periods determined from tide gauge records of the 2004 Sumatra tsunami were used to estimate the size of the tsunami source. A period of 40 minutes dominates the initial motions and spectra are consistent with a fault width of 110 km for a sea depth of 1000 m. Another dominant period of 140 minutes in the spectra observed at the direction of fault azimuth can be explained using a fault length of 1200 km. These values taken together are consistent with source formation on the continental slope with width of 110 km and length of 1200 km along Nicobar and Andaman Islands.

Key words: Dominant period, Sumatra tsunami, tide gauge records.

1. Introduction

The 2004 Sumatra tsunami was generated by an earthquake with origin time of 0 h 58 m 53 s on 26 December 2004 and moment magnitude 9.0, which has epicenter of 3.316°N, 95.854°E, focal depth of 30 km (<http://earthquake.usgs.gov/eqcenter/eqinthenews/2004/usslav/>). The epicenter is located off the west coast of Sumatra, Indonesia. The tsunami propagated across Indian Ocean and resulted in tremendous damages to Indonesia, Thailand, India, Sri Lanka and Maldives. Many Japanese survey teams visited these countries and collected useful data including maximum trace height (e.g. Tsuji *et al.*, 2005). Among the useful data are tide gauge records. Although the main purpose of the tide stations is to record tidal level, the records are also useful to analyze tsunami, especially at long-periods. After the Sumatra tsunami Fujima (2005) carried out the survey in the Maldives and reported that the observed maximum trace height is similar to that of tide gauge record at Hanimaadhoo. This fact suggests that the period was long enough for the tide station to reproduce the sea level.

Here, we will use tsunami tide gauge records to estimate the source size. Yamashita and Sato (1974) derived the theoretical spectrum of tsunami generated at constant depth sea by a rectangular fault under the sea. In this framework the tsunami is formed through unilateral rupture propagation along the fault. Their results reveal a directivity of amplitude distribution. At the same time they show a directivity of dominant period, which is the period corresponding to maximum spectral amplitude. That is, long period components occupy the maximum amplitude along the strike direction and short ones occupy the maximum amplitude in the direction normal to the fault strike. The dominant periods depends on fault parameters. Comparing the observed dominant periods with the theoretical ones at both directions we can estimate fault parameters as a type of inversion

problem.

A preliminary result estimated using inverse refraction diagram was that the source length was 600 km. On the other hand the aftershock area has a linear dimension longer than 1000 km (Satake, 2005). One method to find the tsunami source size without using an inverse refraction diagram is to examine the dominant period in the spectra through analysis of the tide gauge records.

2. Analysis of Tide Gauge Records

As a preliminary step initial periods of tide gauge records are summarized. The tide stations analyzed are as follows: Vishakapatnam, Tuticorin, Kochin, Marmugao (<http://www.nio.org/jsp/tsunami.jsp>), Hanimaadhoo, Male, Gan, Diego Garcia, Salalah, Pt La Rue, Lamu, Zanzibar (<http://ilikai.soest.hawaii.edu/uhsic/>), Showa Base (http://www1.kaiho.mlit.go.jp/KANKYO/KAIYO/jare/tide/tide_index.html), Cocos, Jackson Bay, Arica, Callao (<http://wcatwc.arh.noaa.gov/IndianOSite/IndianO12-26-04.htm>), Sibolga (Tsuji *et al.*, 2005). These tide stations except for Jackson Bay (New Zealand), Arica (Chile) and Callao (Peru) are shown in Fig. 1. The tide gauge data were downloaded and all the data were digitized for use in computer calculations. All the time series are used without correction due to the long period dominance. From the records we measured initial periods, those are time intervals of peak to peak in the first upward motions. The example is shown in Fig. 2 and histogram of the initial periods is shown in Fig. 3. The distribution reveals a dominance of 40 minutes in the period. Dominance of periods shorter than the 40 minutes is possibly attributed to a local oscillation in the vicinity of the station. When we select tide stations located at an azimuth between N280°E and N197°E to the epicenter, which correspond to stations approximately located at a relative azimuth ϕ of 270° as defined in Fig. 1, the average period is 39.4 minutes. This result suggests a dominance of 40 minutes in the spectral components.

The time series were decomposed into amplitude spectra selecting ones of good quality. The tide station of Vishaka-

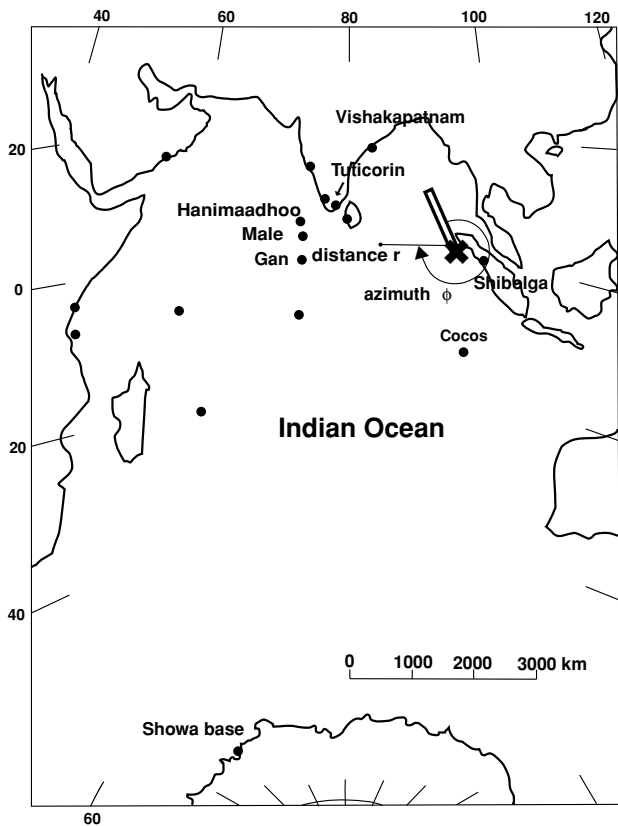


Fig. 1. Tide stations used for spectral analysis and fault source assumed. Cross is epicenter.

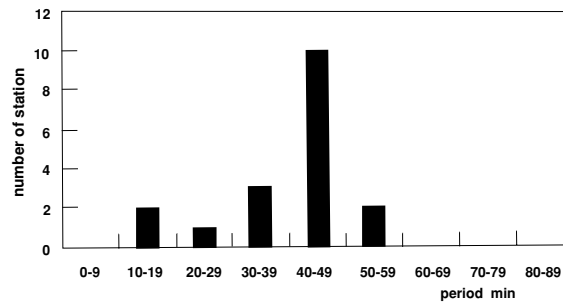


Fig. 3. Classification of initial periods observed at tide stations.

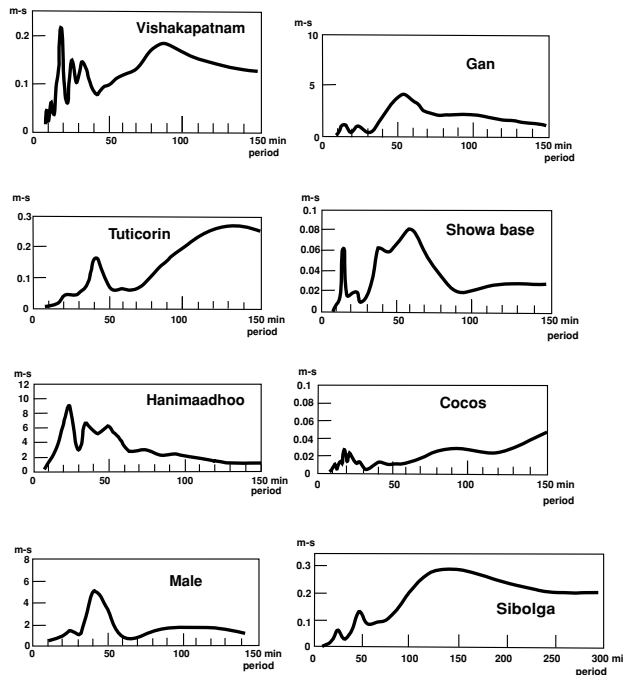


Fig. 4. Amplitude spectra of 2004 Sumatra tsunami.

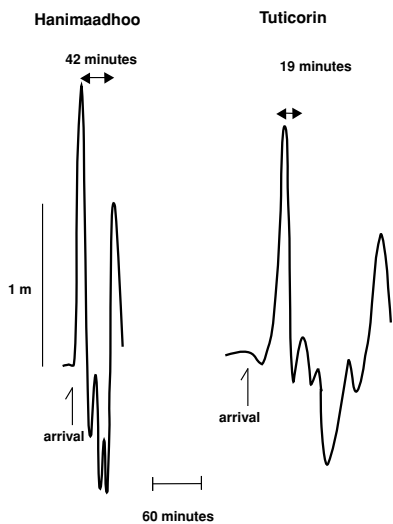


Fig. 2. Examples of reading initial periods.

Table 1. List of tide stations used for spectral analysis. Total sampling times and the time intervals are not same because of differences of original ones.

Tide station	Date (UT)	Time (UT)	Time interval (min)
Vishakapatnam	Dec. 26	3:00–9:00	1
Tuticorin	Dec. 26	2:00–8:00	1
Hanimaadhoo	Dec. 26	4:00–16:00	2
Male	Dec. 26	0:00–24:00	4
Gan	Dec. 26	0:00–18:00	4
Showa base	Dec. 26–27	19:00–1:00	1
Cocos	Dec. 26	3:00–9:00	1
Sibolga	Dec. 26	2:00–14:00	2

patnam was of lesser quality but included in the analysis because of its approximate direction along strike. The discontinuous parts of the record were interpolated. A method of spectral analysis was the same as that of Abe (2005). Thus, spectra for eight tide stations were obtained : Vishakapatnam, Tuticorin, Hanimaadhoo, Male, Gan, Showa base, Cocos, Sibolga. Total sampling time in universal time (UT) and the interval in minute are shown in Table 1 with results shown in Fig. 4. In the figure we find many cases of dominance at 40 min. The representative cases are those

of Tuticorin and Male. The data from Hanimaadhoo shows an indirect dominance of 40 min from the interpretation of splitting into 35 and 49 min. At Showa base a small peak of a 40 min was observed. At Gan a period of 52 min is dominant. Thus, many instances of a 40 min dominant period are also revealed in the spectra. Because the total spectrum is a result of summation during the entire record that is sampled and because the dominant period coincides with the initial period, it suggests no excitation of secondary undulation with other period components. It is reasonable that

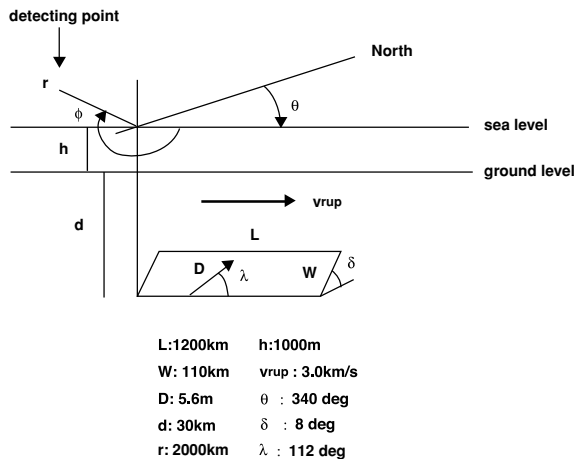


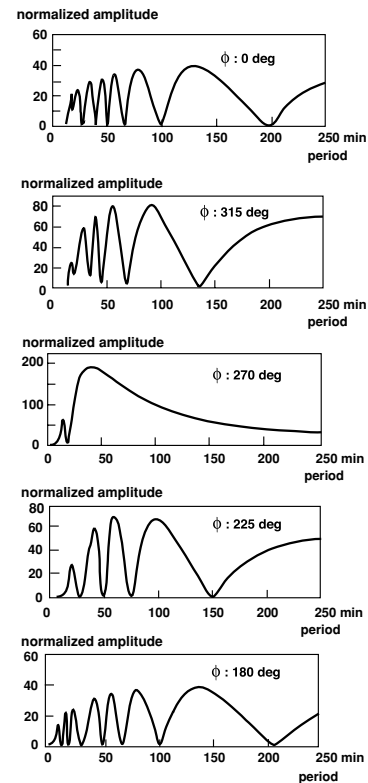
Fig. 5. Definition of fault parameters.

the simple peaks of 40 min observed at Tuticorin and Male reflect the initial condition. To discuss the nature of the source we will use this result as a representative period normal to the direction of the fault.

On the other hand, the spectrum along the strike direction shows a dominance of longer periods. An example is data observed at Sibolga, Indonesia showing a peak in the period of 139 min. The station azimuth is sub-parallel to the strike direction. Dominance of the same period is also found in Tuticorin, which is at a greater azimuth from the strike direction. We will use the period of 139 minutes as a representative period of the strike direction.

3. Theoretical Model

We calculated the theoretical spectrum using the formula by Yamashita and Sato (1974). The fault model and the parameters are illustrated as shown in Fig. 5. The spectrum is estimated at a point on the sea having a distance of r from the origin, which is a corner of the lower margin of the fault plane. The azimuth ϕ is defined relative to the strike direction. Direction of rupture propagation is taken to be positive in the x direction. In the calculation, the fault mechanism is assumed to be the same as derived by Yamanaka (2005), which consists of a seismic moment of 3.5×10^{22} Nm, azimuth angle of N340°E, dip angle of 8 degrees and slip angle of 112 degrees. This solution is typical of that for an inter-plate thrust fault of low dip-angle. Other parameters needed to estimate the spectrum are taken from the original paper except for rupture velocity v_{rup} . Variable parameters are length L , width W , fault depth d , rupture velocity v_{rup} and sea depth h . Dislocation D depends on seismic moment, length and width. A rigidity of 4.3×10^{10} N/m² was assumed to be constant. The criterion to determine the best parameters is that which reproduces the dominant period, defined as the period of the maximum amplitude in the spectrum. In every case the spectrum is estimated at a distance r of 2000 km and relative azimuths ϕ of 180 and 270 degrees. Neglecting the effect of the sea-land boundary at the tide stations we compare the observed spectra with the theoretical ones. The combination of optimal parameters was determined by trial and error, and the best fit model was selected through a comparison of more than 30 models. The

Fig. 6. Theoretical tsunami spectra. Amplitude is normalized by Dh/v_s , in which v_s is S -wave velocity.

amplitude is normalized by hD/v_s and shown for as a function of period, in which v_s is the velocity of S wave. The spectra for relative azimuths of 0, 315, 270, 225 and 180 degrees are shown in Fig. 6. The best fit model consists of the length=1200 km, width=110 km, sea depth=1000 m, rupture velocity=3 km/s and fault depth=30 km. As shown in Fig. 6 we can explain the dominant 40 and 139 minutes at azimuths normal and parallel to strike, respectively.

The dominant period of 93 minute observed at Cocos approximately coincides to the 100 minutes theoretical period calculated at a relative azimuth of 225 degrees. Moreover, a dominant period of 83 minutes observed at Vishakapatnam, is approximately equal to the 90 minutes theoretical period at a relative azimuth of 315 degrees.

The tsunami source was estimated to occur on the continental slope in front of Nicobar and Andaman Islands by Satake (2005). The sea depth there approximately varies from 4000 to 0 m on the slope. Assuming a constant slope from 4000 m to 0 m in the sea depth and calculating the travel time of tsunami across the slope, we obtain the average velocity of 99 m/s for a sea depth of 1000 m. This consideration gives us a physical meaning of the sea depth of 1000 m obtained in the former section. At the same time the width of 110 km is found to be approximately same the horizontal dimension of the slope. This is consistent with the source covering all the sloping area.

4. Parameter Dependence of Dominant Period

Parameter dependence of the dominant period is indicated by the results shown in Figs. 7–9. In these calculations one parameter is variable and others are held constant

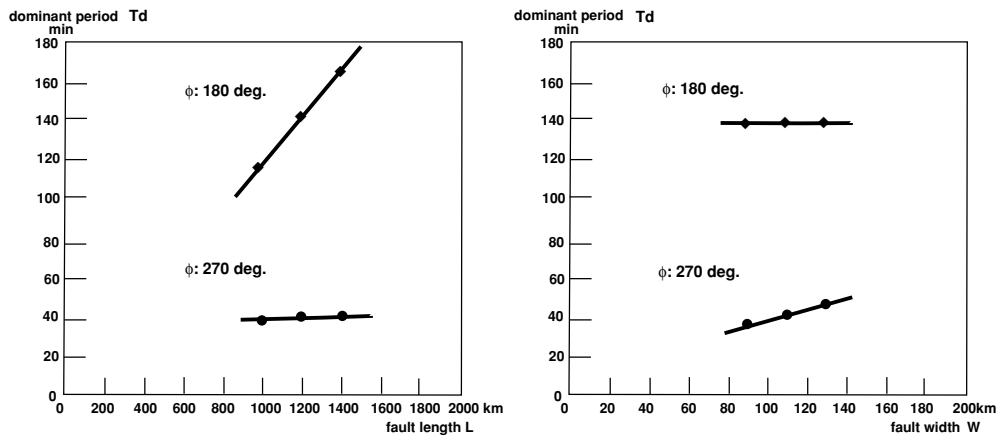


Fig. 7. Dependences of dominant period on fault length L (Top) and width W (Bottom). Other fault parameters are fixed same as ones of the best one.

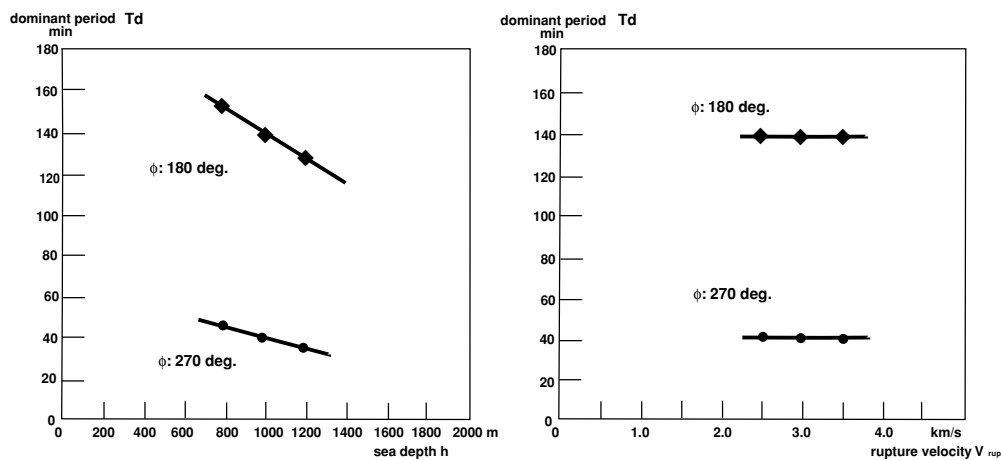


Fig. 8. Dependences of dominant period on sea depth h (Top) and rupture velocity v_{rup} (Bottom). Other fault parameters are fixed same as ones of the best one.

to values determined for the best fit model. The calculations were determined for two positions 2000 km distant from the lower margin and at relative azimuths of 270 and 180 degrees. These points and one for the best fit model are shown in each case. The right point corresponds to an increase and the left one do to a decrease. The slope represents variation of the dominant period with an increase in the parameter. The horizontal line indicates no variation and the solution is locally stable. From this result it is understood that the length and width give large effects to the dominant periods at azimuth angles of 180 and 270 degrees, respectively (Fig. 7). Sea depth can cause a variation of the dominant period for both the directions (Top of Fig. 8). On the other hand fault depth results in a small effect to the dominant period at the direction of 270 degrees (Fig. 9). Effect of rupture velocity to the dominant period is small (Fig. 8). Thus, it is concluded that length, width and sea depth are the critical parameters that determines the dominant periods of tsunami.

5. Discussion

It is interesting to discuss the relation between wave length of a tsunami and fault size. In the direction of fault strike, wave length corresponding to the dominant period

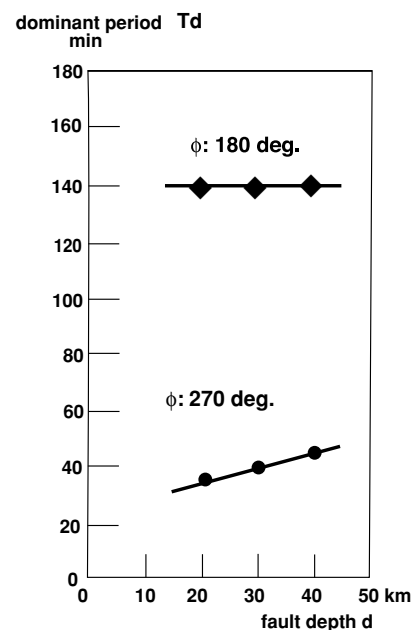


Fig. 9. Dependences of dominant period on fault depth d . Other fault parameters are fixed same as ones of the best one.

of 139 min is 826 km for a sea depth of 1000 m in depth, which is 69% (approximately 2/3) of the fault length, 1200 km. In the case of a tsunami consisting of one wave length corresponding to the fault length the period is equal to 202 minutes. In the normal direction of the fault wave length corresponding to the dominant period of 40 minutes for a sea depth of 1000 m is 238 km which is 216% (approximately 2.0) of the width, 110 km. Since wave length is approximated as $\lambda_W = 2W / ((2n + 1) \cos \delta)$ in which n is positive integer, the fundamental mode of fault width was excited and observed as the dominant period.

Lay *et al.* (2005) proposed a model consisting from three fault segments distributed along the island arc based on the rupture process. The total length reaches about 1300 km and the average rupture velocity of 2.5 km/s was estimated. Our result is consistent with their result.

6. Conclusion

We studied dominant periods of the 2004 Sumatra tsunami from the spectra of tide gauge records. As a result dominant periods of 40 and 139 minutes were obtained in the normal direction and parallel direction of the fault, respectively. A theoretical approach was used to explain the relation between dominant period and fault size. The optimal source parameters obtained from this approach are a 1200 km fault length, a 110 km fault width and a sea depth of 1000 m. The dominant period of 40 minutes is explained from the fundamental mode of fault width. This result does not conflict with a source region located on the continental slope.

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